Ph.D. Qualifying Examination
Department of Astronomy
May 28, 2020
11:00 a.m. — 4:00 p.m.

Name: ________________________

Student Number: ________________

The exam sheets are inside this envelope and are not fastened together. When you are finished, please put the questions and your answer sheets back in the envelope in the correct order. **Be sure the student number given to you by the proctor is on every page of your answers.**

Students pursuing the Astronomy PhD or students pursuing the Astrophysics PhD and taking only the Astronomy qualifying exam **MUST** answer the **FIRST TWO** questions and **SIX** more questions from the remaining **EIGHT**. These students have 5 hours to complete the exam and must finish by **4:00 p.m**.

Students pursuing the Astrophysics PhD who are meeting their qualifying exam requirement by taking part of the Physics qualifying exam and part of the Astronomy qualifying exam must answer **ONE** of the **FIRST TWO** questions and **FOUR** more questions from the remaining **EIGHT**. These students must finish by **2:30 p.m.** (3.5 hours).

M.A. students must do **ONE** of the **FIRST TWO** problems and **THREE** more problems from the remaining **EIGHT**. M.A. students must finish by **1:30 p.m.** (2.5 hours).

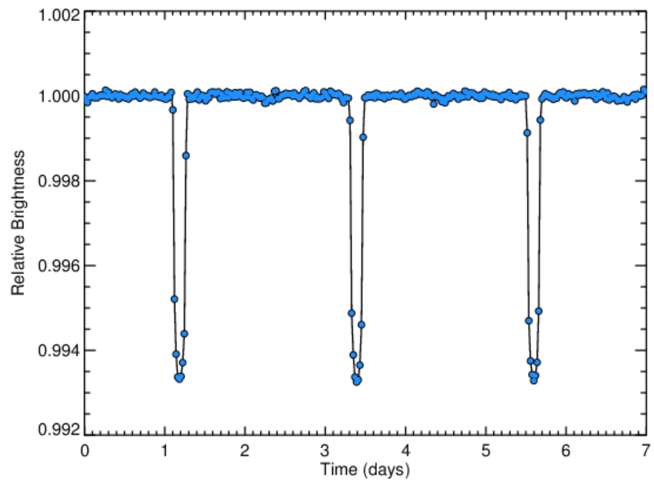
Physical Constants

$c = 3.00 \times 10^{10}$ cm/s
$G = 6.67 \times 10^{-8}$ dyn cm^2/g^2
$h = 6.63 \times 10^{-27}$ erg s
$k = 1.38 \times 10^{-16}$ erg/K
$m_p = 1.67 \times 10^{-24}$ g
$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}
$e = 4.80 \times 10^{-10}$ esu
$m_e = 9.11 \times 10^{-28}$ g
$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
$\sigma_T = 6.65 \times 10^{-25}$ cm^2
1 eV = 1.6×10^{-12} erg

$R_\odot = 6.96 \times 10^{10}$ cm
$M_\odot = 1.99 \times 10^{33}$ g
$L_\odot = 3.90 \times 10^{33}$ erg/s
A.U. = 1.50×10^{13} cm
1 year = 3.16×10^{7} s
1 parsec = 3.09×10^{18} cm
$M_{V\odot} = 4.83$ mag
$B.C._\odot = -0.07$ mag
$(B-V)_\odot = 0.64$ mag
$T_{eff,\odot} = 5770$ K
$M_E = 5.97 \times 10^{27}$ g
$R_E = 6.38 \times 10^{8}$ cm

REQUIRED QUESTION 1. Exoplanets

An exoplanet has been discovered using the transit method. The light curve is shown below.



The host star is a K2 V star with a mass of 0.82 solar masses and a radius of 0.735 solar radii. The orbital period is 2.23 days.

Follow-up spectroscopy indicates that the radial velocity of the host star also varies with the same period, with a half-amplitude of 30 m s^{-1}.

Be sure to use appropriate significant digits in your responses.

A. What is the radius of the exoplanet in km? [20 pts]

B. What is the semi-major axis of the planet's orbit in AU and km? [20 pts]

C. What is the mass of the exoplanet? [20 pts]

D. What is the density of the exoplanet? [20 pts]

E. What is the likely composition of the exoplanet? [20 pts]

REQUIRED: QUESTION 2. Star Clusters

Color-magnitude diagrams for two star clusters are shown on the following two pages. **Please include these pages with figures (with your work shown) in your assembled answers.**

A. For each of these clusters, identify and label the following major evolutionary stages: main-sequence, main sequence turnoff, subgiant branch, red giant branch, and horizontal branch. [25 pts]

B. For each of these stages, describe:

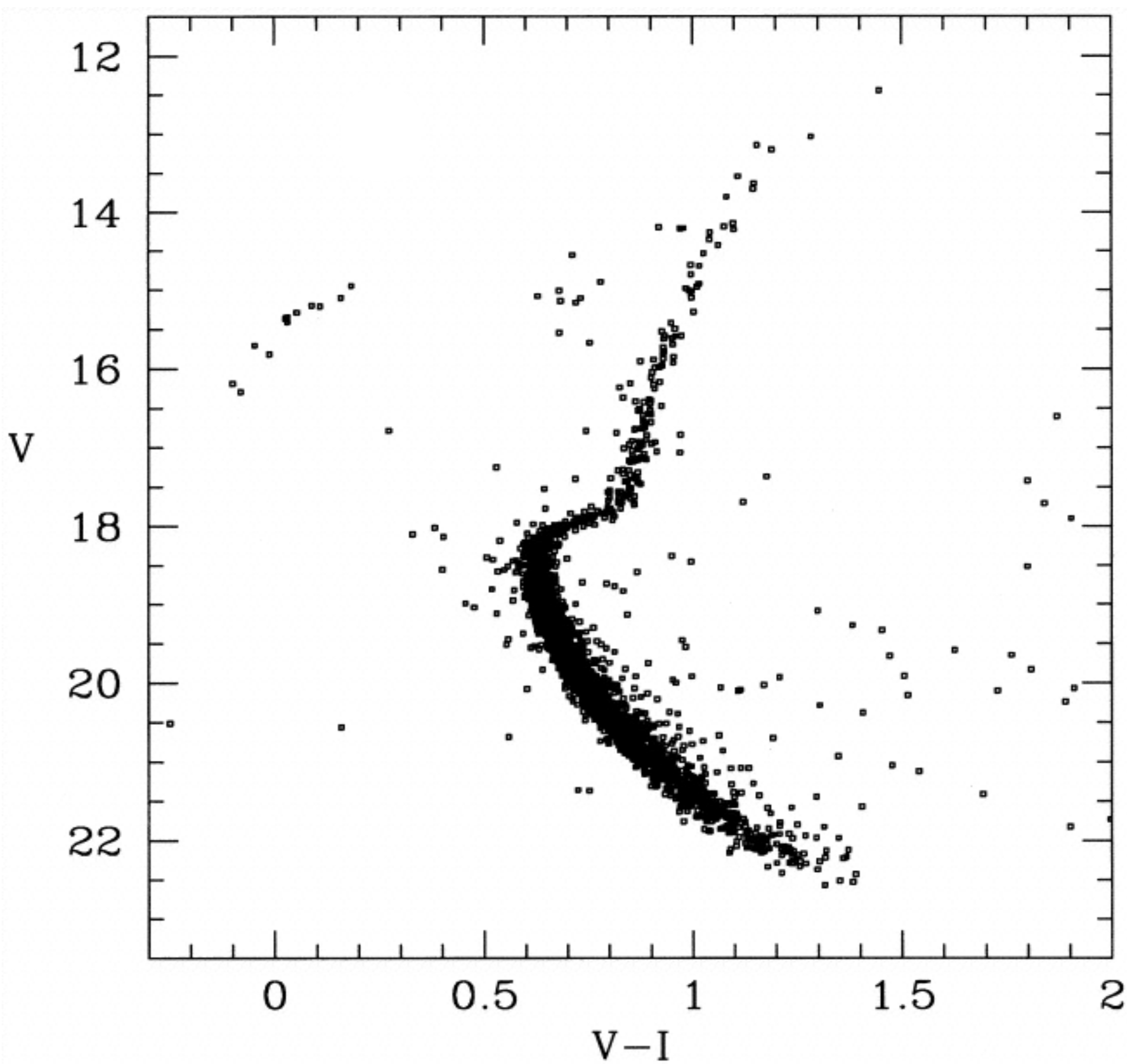
   (i) the dominant form(s) of energy generation;
   (ii) relevant nuclear reactions, and where in the star they occur;
   (iii) where in the star (major) convective and radiative regions occur.

   You might find that some stages require further subdivision, for example, not all main sequence stars have the same dominant nuclear reactions. [45 pts]

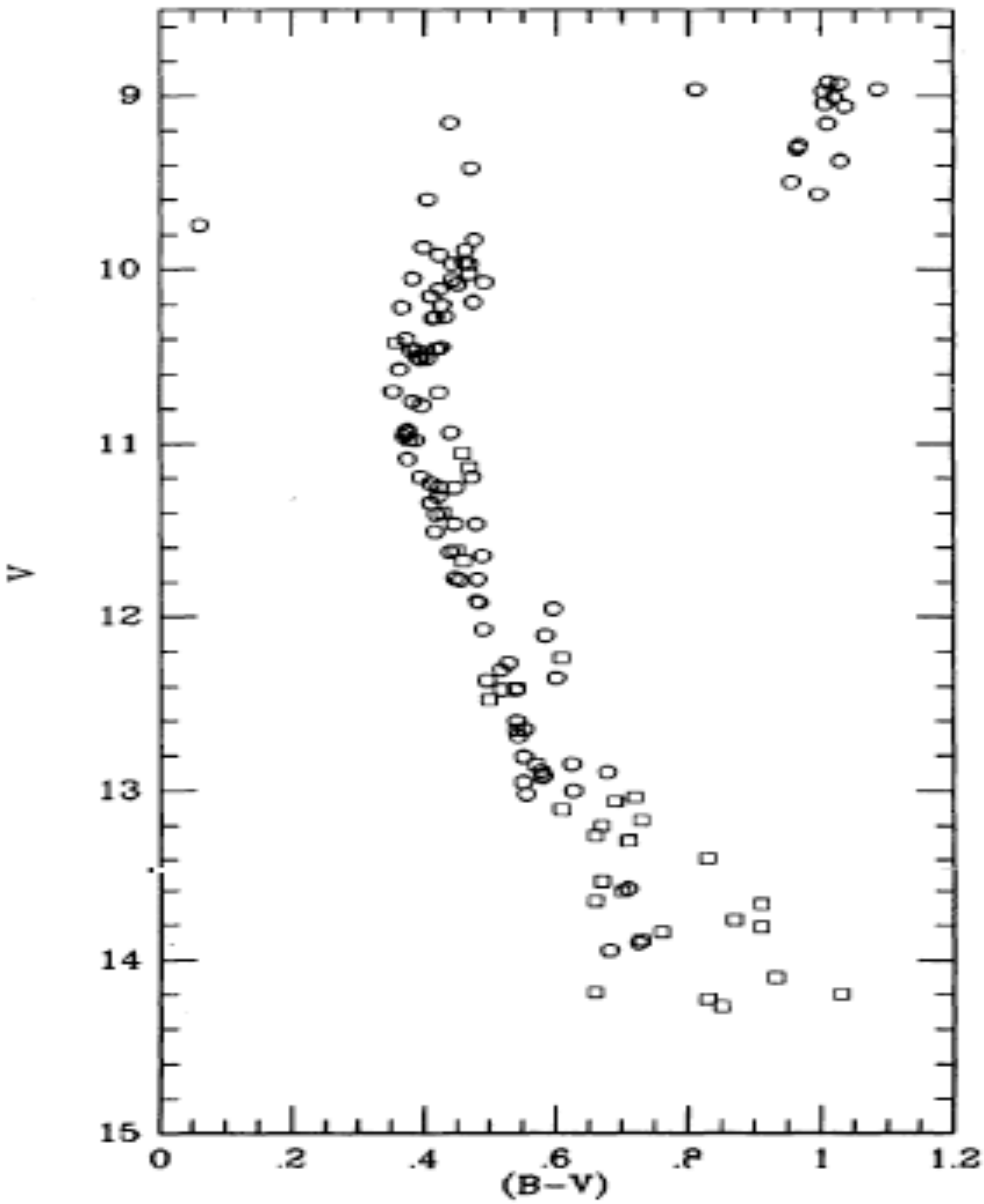
C. On the diagrams, indicate where the following kinds of stars would be, even if they are not present in the cluster and even if they fall outside the boundaries of the plot: RR Lyraes, Cepheids, white dwarf stars, blue stragglers. [10 pts]

D. Give an estimate of the cluster age with an associated uncertainty, explaining clearly what features you used and your reasoning to derive the age. What additional information would you need to have to derive an accurate age for the cluster? [20 pts]

Figures for Question 2 **Student Number _________**



Figures for Question 2 **Student Number ________**



QUESTION 3. Cosmology

The aim of the question is to highlight some of the properties of the expansion dynamics of our universe, in which the dark energy exists, and may very well be in the form of the cosmological constant. Consider only a flat universe with $H_0 = 70$ km s^{-1} Mpc^{-1}, $\Omega_{m,0} = 0.3$ and $\Omega_{\Lambda,0} = 0.7$.

A. Find the value of the critical density today ($\rho_{crit,0}$) in cgs units, and from it the densities of matter and of Λ. Note that they are comparable today. [10 pts]

B. At what redshift were the two densities identical (use subscript 'eq')? [10 pts]

C. How many orders of magnitude separated the two densities at the time of photon decoupling (use subscript 'dec')? [10 pts]

D. The reason why many cosmologists prefer a dynamical dark matter instead of the cosmological constant is the so called fine-tuning problem. To illustrate it, suppose the matter density and Λ were similar already at the time of decoupling? What would be $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ today? [10 pts]

E. Discuss some of the implications of such a universe. [5 pts]

F. Based on the above, how would the dynamical dark matter help resolve the fine-tuning problem? [10 pts]

G. Show that in a Λ universe the Hubble parameter tends to a constant value H_{lim}. What is this value in km s^{-1} Mpc^{-1}? [10 pts]

H. Explain how it is possible that the universe is accelerating even though the Hubble parameter stays the same. Why is there no conflict there? [10 pts]

I. M101 lies at a distance of 6.5 Mpc, our cosmic neighborhood, and can be seen even with a 4" telescope. A "weird" feature of the accelerating universe is that one day there will be very few galaxies that could be seen even with WIYN 3.5 m. For simplicity, assume that we are already in Λ-only expansion regime, with the Hubble constant from part G (if you haven't solved part G, use today's H_0). In how many Gyr from now will M101 be at $z = 1$? (Hints: Derive the change of the scale factor first. Derive the answer in Hubble times as an intermediate step.) [20 pts]

J. In some models of dark energy, the universe experiences a Big Rip, where even the atoms will be ripped apart by the expansion. What sort of equation of state (w) do such models possess? [5 pts]

Potentially useful formulae:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -4\pi G\left(\frac{\rho}{3} + \frac{p}{c^2}\right) + \frac{\Lambda}{3}$$

QUESTION 4. Galaxies

Consider a hypothetical galaxy in the Virgo Cluster (distance = 16.5 Mpc) with m_B = 14.53 mag, A_B = 0.07 mag, and the centroid of the Mg absorption line complex corresponding to a recessional velocity of -305 km/s. Be sure to use appropriate significant digits in your responses.

A. Calculate the distance modulus for this galaxy, assuming it is near the cluster core. Using this value, calculate the absolute B-magnitude of this galaxy (M_B). Show your work. [10 pts]

B. Calculate the peculiar velocity for this galaxy. Show your work and be explicit about any assumptions you have made. Evaluate the value you have derived: is it within the range you would expect for a galaxy in a cluster (e.g., provide an expected range for peculiar velocities of galaxy clusters). [10 pts]

C. Stellar population modeling of this galaxy indicates two dominant stellar populations, one with a current M_B of -16.32 and B-V color of 0.76 and a second with current M_B of -15.12 and B-V color of 0.95. Based on this information, what is the M_V and B-V color of the galaxy? Be sure to start from first principles regarding the definition of a magnitude and show your work. Note that it may be advantageous to calculate M_V for each population before calculating M_V for the galaxy. [25 pts]

D. Given the information available, provide a justification for whether this galaxy is likely to be an early-type or late-type galaxy. Include in your answer whether or not you think this may be a dwarf galaxy and your justification for that conclusion. What further observational evidence could you obtain that would help you distinguish between an early-type or late-type galaxy? [15 pts]

E. Describe the physical processes that allow the Baldwin, Phillips, & Terlevich (BPT) diagram to be a good diagnostic for determining whether or not an observed source is an HII region or an AGN. Include a sketch of the BPT diagram, with the axes labeled and fiducial values, and the approximate locations of HII regions and AGN noted. [20 pts]

F. Provide a sketch illustrating the observed morphology-density relation, including trends for ellipticals, spirals, and S0 galaxies, including axis labels and fiducial values. Describe the possible physical processes that could drive this relation. [20 pts]

QUESTION 5. Galactic Dynamics

A. Consider a gravitational potential with the following form:

$$\phi(r) = \frac{-GM_0}{a} \ln\left(\frac{a+r}{r}\right)$$

where M_0 is the total mass and a is a radial scale parameter. Compute the following relations for this model:

(i) the mass contained within radius $r, M(r)$
(ii) the rotation curve, $v_c(r)$
(iii) the density profile, $\rho(r)$.

Next, find limiting expressions for $M(r)$ in the limits $r \ll a$ and $r \gg a$. Show that the rotation curve approaches a flat form for $r \ll a$ and a Keplerian form for $r \gg a$. What does the Keplerian form of the rotation curve for $r \gg a$ say about the mass distribution? Also find limiting expressions for $\rho(r)$ in the limits $r \ll a$ and $r \gg a$. Is there anything unphysical about the behavior of $\rho(r)$ in the limit $r \to 0$?

Sketch the behavior of the radial profiles (i), (ii), and (iii) using your small and large r results from above. Use log-log axes for the density profile plot. Indicate the approximate location of a on each of the plots. [50 pts]

The following formulas may be useful.

$$\frac{d\phi(r)}{dr} = \frac{GM(r)}{r^2}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\nabla^2 \phi = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = 4\pi G\rho$$

$$\int \frac{1}{(1+x)^2} dx = -\frac{1}{(1+x)}$$

B. The one-dimensional velocity dispersion, $\sigma(r)$, for a system with the following gravitational potential

$$\Phi(r) = -GM/[b(1+r^2/b^2)^{0.5}]$$

is

$$\sigma(r) = (-1/6\, \Phi(r))^{0.5}.$$

(i) Write the parameter used to define 'soft' and 'hard' binaries. Next, calculate the semi-major axis of a binary at the threshold between 'soft' and 'hard' binaries for both r=0 and r=b for a system with $M=10^5\ M_{sun}$ and b=1 pc (assume stars have all the same mass $m=0.5\ M_{sun}$).

(ii) Discuss the difference between the evolution of soft and hard binaries in a star cluster.
[30 pts]

C.

(i) Provide an example of a collisionless stellar system and one of a collisional stellar system.

(ii) Discuss the main aspects (physical processes, timescales) of the dynamics of collisional stellar systems. [20 pts]

QUESTION 6. The Interstellar Medium

Consider the case of an ionized gas with a specific electron temperature (T_e) and electron density (n_e). Answer the following, assuming that all upward transitions in the relevant atoms are due to collisional excitation.

A. Specify a specific ionization state for a specific type of metal atom that can be used to measure the electron density (n_e) of the gas. There are multiple correct answers to this question – just describe one such example. [10 pts]

B. Draw the energy level diagram for the metal ion you specified in A. Include the three lowest-lying energy states only and include the term and level designations. Label the downward transitions from the upper state(s) to the lower state(s) with the wavelengths of the astrophysically relevant transitions essential for the measurement of the electron density. [15 pts]

C. Explain how the electron density (n_e) can be inferred from the emission-line flux ratio of specific lines shown in your energy level diagram in B. That is, give a detailed explanation of the physics behind the measurement of n_e using the appropriate set of emission lines. Make it clear why this particular set of emission lines is density sensitive. Give characteristic values for the ratio and the corresponding densities in the low- and high-density limits. [25 pts]

D. Specify a specific ionization state for a specific type of metal atom that can be used to measure the electron temperature (T_e) of the gas. There are multiple correct answers to this question – just describe one such example. [10 pts]

E. Draw the energy level diagram for the metal ion you specified in D. Include the five lowest-lying energy states only and include the term and level designations. Label the downward transitions from the upper state(s) to the lower state(s) with the wavelengths of the astrophysically relevant transitions essential for the measurement of the electron temperature. [15 pts]

F. Explain how the electron temperature (T_e) can be inferred from the emission-line flux ratio of specific lines shown in your energy level diagram in E. That is, give a detailed explanation of the physics behind the measurement of T_e using the appropriate set of emission lines. Make it clear why this particular set of emission lines is temperature sensitive. Give characteristic values for the ratio (approximate values are fine) and the corresponding temperatures that are observed in real nebulae. [25 pts]

QUESTION 7. Observational Astronomy

Suppose you have been allocated one night of observing time to acquire imaging data with a ground-based telescope equipped with a typical research-grade, low-noise CCD camera. Your scientific objective is to carry out absolute ("all-sky") photometry of the individual stars in a nearby star cluster with a standard set of Johnson UBV filters. The CCD detector has a 16-bit A/D converter, a gain setting of 1 e-/ADU, and uniform sensitivity across all three filters.

You will use the data you acquire to quantify the magnitudes and colors for the individual resolved stars in the cluster. The stars have a wide range of apparent magnitudes: the faintest stars you are targeting are 7.5 magnitudes dimmer than the brightest stars. The brightest stars in the cluster just barely saturate the CCD in a 10-second exposure in each filter. You need to achieve at least "1% photometry" over this entire magnitude range in order to accomplish your science goals. The area of sky you are targeting has not been included in other photometric surveys, so you will need to acquire your own standard star observations. Assume that the sky conditions on the night in question are clear and stable ("photometric").

A. **List and describe all calibration observations you will need to acquire in order to reduce your imaging data and accomplish calibrated absolute photometry of the stars in the cluster. Specify the type and purpose of each set of calibration observations and explain exactly how and when you would acquire the data.** For each set of calibrations, include detailed information about: when (what part of the day or night) you would acquire the data; which filter(s) you would use; what observing strategy you would employ (how many exposures of each type do you need? What is the cadence of the observations?); and any general information about integration time, as appropriate. **In all cases, briefly justify the reasons for your choices** (i.e., do not simply list the required data without giving an explanation). [70 pts]

B. **Next, specify your chosen observing strategy for the object observations.** What observations do you need to acquire, and how do you need to acquire them, in order to achieve at least 1% photometry given the target objects, observing conditions, and equipment setup specified above? Fully explain your answer and include an estimate of the required exposure times and the sequence and number of exposures. [30 pts]

QUESTION 8. Simple Models of Stars

In this problem, consider models of spherically symmetric, chemically homogeneous stars. You may assume that radiation pressure is negligible compared to gas pressure, and that the ideal gas law is a good approximation for the equation of state; assume it has the following form:

$$P_r = \frac{R_g \rho_r}{\mu} T_r \ ,$$

where r is the radial coordinate and subscript "r" means the quantity is evaluated at r; P_r, ρ_r, and T_r are the pressure, density, and temperature (at r); μ is the mean molecular weight; and R_g is the gas constant. You may also need the following two equations of stellar structure:

$$dM_r = 4\pi r^2 \rho_r dr \ , \qquad dP_r = -G\rho_r \frac{M_r}{r^2} dr \ .$$

A. Consider the model with constant density, i.e. $\rho_r = \rho_c = \rho_{avg}$, where subscripts "c" and "avg" mean central and average. Although unrealistic, such models begin to yield insight. Using the above equations, and integrating where necessary, derive expressions for $M(r)$, P_r, and T_r, in terms of *only* the total mass, M, the total radius, R, and physical constants (such as G); $M(r) \equiv M_r$ is the total mass from the center to r. You will need to assume "zero" surface boundary conditions (that is, $P(R) = 0 = T(R)$), and since we don't know the central pressure, you will need to integrate the equation of hydrostatic equilibrium from the surface to r (instead of from the center to r). [35 pts]

B. Consider the slightly more realistic "linear" model with $\rho_r = \rho_c \left(1 - \frac{r}{R}\right)$.

Use the equation of mass continuity (from above) to derive an expression for ρ_c in terms of *only* M and R (and possibly some constants).

Compare ρ_c between the linear and constant-density models: which model is more centrally concentrated? [30 pts]

C. If we derive the pressure for the linear model in the same way as we did for the constant-density model in part A, we would find,

$$P_r = \frac{5}{4\pi} G \left(\frac{M}{R^2}\right)^2 \left[1 - \frac{24}{5}\left(\frac{r}{R}\right)^2 + \frac{28}{5}\left(\frac{r}{R}\right)^3 - \frac{9}{5}\left(\frac{r}{R}\right)^4\right].$$

Using this, and the result from part A, show that the *central* pressure of the linear model is larger than that of the constant-density model, and explain the physical reasons for this.

How do you expect the central temperature of the linear model to compare to that of the constant-density model? [35 pts]

QUESTION 9. Stellar Remnants

Consider the three possible remnants of stellar evolution: white dwarfs, neutron stars, and black holes.

A. For each type of object, specify the physical properties (e.g., mass, radius, composition, physical state), and describe how the object forms. Include the expected mass range of the stars on the Main Sequence that will end up as a white dwarf, neutron star, or black hole. Also include (as appropriate) explanations of such terms as the Chandrasekhar limit, the white dwarf mass-radius relation and the cooling time scale, electron degeneracy, neutron degeneracy, pulsars, and the Schwarzschild radius. [50 pts]

B. Clearly explain how each of these three types of stellar remnant can be studied observationally. Include detailed information about the wavelength region (e.g., X-ray, visible, radio) and observational methods that are most useful for the detection and investigation of each object type. [50 pts]

QUESTION 10. Fast-Spinning White Dwarf

Reding et al. (2020) have reported the discovery of a fast-spinning, isolated white dwarf with a rotational period of 317.278 ± 0.013 seconds. The star has a parallax of 12.94 ± 0.11 mas and proper motion of (49.6, -40.0 mas yr^{-1}), and a bolometric magnitude of 17.37. The authors determined its temperature to be 8237 ± 206 K and its surface gravity to be log g = 8.09 ± 0.05 (g in cgs units). They estimate its mass at 0.65 solar masses and its age to be about 2 Gyr. Be sure to use appropriate significant digits in your responses.

A. Compute the white dwarf's luminosity in solar units. [20 pts]

B. Estimate the white dwarf's radius. [20 pts]

C. Estimate the white dwarf's rotational velocity at the equator. [20 pts]

D. Compute the centrifugal acceleration at the equator of the white dwarf. How does it compare to the gravitational acceleration at the equator? Recall that the formula for computing centrifugal acceleration is $a_c = v^2/r$. [20 pts]

E. What would the rotation period of this white dwarf need to be for the centrifugal acceleration to equal the gravitational acceleration? [20 pts]