Ph.D. Qualifying Examination
Department of Astronomy
May 24, 2019
10:00 a.m. — 3:00 p.m.

Name: ___________________________

Student Number: _________________

The exam sheets are inside this envelope and are not fastened together. When you are finished, please put the questions and your answer sheets back in the envelope in the correct order. **Be sure the student number given to you by the proctor is on every page of your answers.**

Students pursuing the Astronomy PhD or students pursuing the Astrophysics PhD and taking only the Astronomy qualifying exam **MUST** answer the **FIRST TWO** questions and **SIX** more questions from the remaining **EIGHT**. These students have 5 hours to complete the exam and must finish by **3:00 p.m**.

Students pursuing the Astrophysics PhD who are meeting their qualifying exam requirement by taking part of the Physics qualifying exam and part of the Astronomy qualifying exam must answer **ONE** of the **FIRST TWO** questions and **FOUR** more questions from the remaining **EIGHT**. These students must finish by **1:30 p.m.** (3.5 hours).

M.A. students must do **ONE** of the **FIRST TWO** problems and **THREE** more problems from the remaining **EIGHT**. M.A. students must finish by **12:30 p.m.** (2.5 hours).

Physical Constants

$c = 3.00 \times 10^{10}$ cm/s
$G = 6.67 \times 10^{-8}$ dyn cm^2/g^2
$h = 6.63 \times 10^{-27}$ erg s
$k = 1.38 \times 10^{-16}$ erg/K
$m_p = 1.67 \times 10^{-24}$ g
$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}
$e = 4.80 \times 10^{-10}$ esu
$m_e = 9.11 \times 10^{-28}$ g
$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
$\sigma_T = 6.65 \times 10^{-25}$ cm^2
1 eV = 1.6×10^{-12} erg

$R_\odot = 6.96 \times 10^{10}$ cm
$M_\odot = 1.99 \times 10^{33}$ g
$L_\odot = 3.90 \times 10^{33}$ erg/s
A.U. = 1.50×10^{13} cm
1 year = 3.16×10^{7} s
1 parsec = 3.09×10^{18} cm
$M_{V\odot} = 4.83$ mag
$B.C._\odot = -0.07$ mag
$(B-V)_\odot = 0.64$ mag
$T_{eff,\odot} = 5770$ K
$M_E = 5.97 \times 10^{27}$ g
$R_E = 6.38 \times 10^{8}$ cm

REQUIRED: QUESTION 1. Fiducial Values

An important aspect of being an effective scientist is to be able to recognize when preliminary results are likely to be incorrect. The following questions are designed to demonstrate your mastery of fiducial values in astronomy.

A. Using a consistent set of units (i.e., all in meters or all in centimeters), what are the approximate sizes of the following 8 objects: the Sun, the Milky Way Galaxy, a carbon nucleus, a terrestrial planet in the Solar System, an adult human, a cluster of galaxies, the Solar System, a molecular cloud in the Galaxy? [16 pts]

B. Using a consistent set of units (i.e., all in kilograms or all in solar masses), what are the approximate masses of the following 8 objects: the Sun, the Milky Way Galaxy, a carbon nucleus, a terrestrial planet in the Solar System, an adult human, a cluster of galaxies, the Solar System, a molecular cloud in the Galaxy? [16 pts]

C. What are the typical B-V colors, effective temperatures (T_{eff}), and absolute magnitudes (M_V) for the following three types of stars: O-stars, G-dwarfs, K-giants? [18 pts]

D. What are the currently accepted values for T_{CMB}, H_0, Ω_{baryon}, Ω_{DM}, and Ω_{Λ}? [10 pts]

E. Sketch the typical metallicity distribution as a function of radius for HII regions in late-type spiral galaxies. Be sure to label your axes and indicate the appropriate approximate physical values for both the linear scale and metallicity indicator. [15 pts]

F. Consider the case of a low mass galaxy with solid body rotation. Sketch the rotation curve as a function of radius. Be sure to include appropriate values on the x and y axes of your sketch. [15 pts]

G. Sketch the generic form of the luminosity function for galaxies. Note major features in your sketch and clearly denote 3-4 well-spaced representative values on the x-axis. [10 pts]

REQUIRED: QUESTION 2. Supermassive Black Hole in M87

In this question you will use simple physical arguments to derive some basic properties of the supermassive black hole (SMBH) in M87: its mass, physical and angular Schwarzschild radii, accretion rate, as well as determine the observability of its "shadow".

A. The recently released radio interferometric image of a black hole shadow brought SMBHs into the spotlight of astronomers and public alike, as it is the most solid evidence for the existence of such objects and represents a new way of studying them. This is the culmination of efforts that started in 1978, when the first indication of a SMBH in M87 (or any galaxy) was published, based on high values of stellar velocity dispersions. Specifically, Sargent et al. (1978) measured $\sigma = 400$ km/s at $R = 1.5$ arcsec from the center of M87. M87 lies at a distance of $d_{M87} = 16.5$ Mpc. Produce an estimate of SMBH mass (i.e., mass enclosed in 1.5 arcsec) from the data given above. You should use a simplifying assumption that the velocity dispersion is similar to a circular orbital velocity. Give your answer in grams and in solar masses. [25 pts]

B. What is the Schwarzschild radius r_S of this black hole in AU? Derive the expression for r_S from an expression for the escape velocity. Also give the radius in μas (R_S). Partial credit given if you use an appropriate scaling instead. [25 pts]

C. Because of the gravitational bending effects, the observed angular diameter (not the radius) of the ring around the BH shadow (D_{shadow}) is ~5 times the Schwarzschild radius. In an interferometric array, the angular resolution of the array depends on the separation between the telescopes in the same way that it would depend on the diameter of a single radio dish (or telescope objective). What separation d (in km) is needed to resolve the ring (match the resolution to the size)? How does it compare with the Earth's diameter? Observations take place at $\lambda = 1.3$ mm. You can use the formula or scale from the fact that a 4.5-inch telescope has an optical resolution of ~1 arcsec. D_{shadow} was eventually measured to be 42 μas. Use simple scaling to find what it implies for the actual SMBH mass. [25 pts]

D. M87 is an AGN (radio galaxy) with the jets visible even in the optical. The total bolometric luminosity of the AGN core (without galaxy-scale jets) is $L_{bol} = 2.7 \times 10^{42}$ erg/s. Assuming an AGN converts 10% of the mass that falls into it into energy, calculate the accretion rate, first in g/s, then solar masses per year. How does the answer compare to the average accretion in the past, assuming M87 is as old as the universe and assuming the mass from part C above. [25 pts]

QUESTION 3. Observational Astronomy

The intensity values in the pixels of an unprocessed CCD image will include detector-related signatures. In order to prepare a CCD image for astronomical measurements, one typically carries out a series of digital processing steps that correct for and/or remove these instrumental signatures. The commonly used terms for these image processing steps are bias (or zero) correction, dark correction, and flat-field correction.

A. Explain in detail the process of bias correction, including why it is done, what exactly the instrumental bias signature represents, and how the correction is carried out. In addition, *explain exactly why* unprocessed CCD images possess a bias signal. [20 pts]

B. Explain in detail the process of dark correction, including why it is done, what the instrumental dark signature represents, and exactly how the correction is carried out. [20 pts]

C. Explain in detail the process of flat-field correction, including why it is done, what attributes of the unprocessed image the flatfield is correcting for, and how the correction is carried out. Confine your answer to the case of dome flats but be complete in your explanation of what this part of the processing is correcting for. [20 pts]

D. Consider the intensity of a specific pixel in an unprocessed science image as well as the formal uncertainty associated with this intensity value. Explain how the processing associated with the removal of the three instrumental signatures mentioned above will alter the uncertainty in the intensity value for this pixel. Will the formal uncertainty increase or decrease as a result of the processing? Explain your answer. [16 pts]

E. (i) Derive a general relationship for the uncertainty σ_{AVG} that would apply if one averaged together N images, each with an uncertainty of σ_I, where the values of σ_I are not necessarily identical. You may ignore any co-variance terms in your analysis. [10 pts]

(ii) Now simplify your relationship from part (i) for the case where the uncertainties in the individual images (we can call them σ_B) are all the same. [7 pts]

(iii) Imagine that you average together 25 bias images. If the readout noise of a single bias image is 10 e^-, what is the uncertainty (noise) in the composite averaged bias image? [7 pts]

QUESTION 4. Stellar Spectra

Answer each of the questions below with one or two sentences. No essays!

A. The Hydrogen Balmer lines are strongest among stars of spectral type A. Why do they become weaker in both hotter and cooler stars? [10 pts]

B. Describe how the Balmer lines change with luminosity at a fixed temperature. Explain why this is so. [15 pts]

C. What causes the emission in the cores of the Ca II H & K lines at 3933 and 3968 Angstroms? [20 pts]

D. What are the dominant sources of opacity in the atmospheres of O stars, F stars, and M stars? [25 pts]

E. The ground-state spectral lines of Li, Na, K, Rb become really, really strong in cool dwarf stars of spectral types M and L. Why, since these species all have relatively low abundances? [20 pts]

F. For stars and sub-stellar objects with spectral types later than about M6, the presence of Li spectral lines can be used to distinguish whether an object is a star or a brown dwarf. Explain why this works. [10 pts]

QUESTION 5. The Interstellar Medium

An important assumption / approximation that is often applied in the ISM is that of kinetic equilibrium.

A. State the fundamental criterion/criteria that must apply for kinetic equilibrium to be in effect. What is the term usually used to describe gas that is in kinetic equilibrium? [20 pts]

B. Describe the process by which an ensemble of particles not initially in kinetic equilibrium can reach a state of kinetic equilibrium within the ISM. Be as specific as possible. [15 pts]

C. A key related concept is that of relaxation time. Explain what astronomers mean when they refer to the relaxation time of particles in the ISM. [15 pts]

D. Without necessarily carrying out a full derivation, describe the process by which one would determine the relaxation time of various groups of particles in the ISM. Give enough detail to show that you understand the key physical processes involved. [20 pts]

E. Assume that a non-equilibrium population of protons and electrons is created in the ISM. Will the protons reach a state of kinetic equilibrium faster or slower than the electrons? Explain your answer. [15 pts]

F. Gas in the ISM will only reach kinetic equilibrium if the relaxation time is short compared to other timescales for changes in the gas. One such timescale is the gas cooling timescale. Describe the physical process by which the cooling takes place and comment on the relative lengths of the relaxation and cooling timescales. [15 pts]

QUESTION 6. Dust

A. Estimate the temperature of a grain of silicate dust at a distance of 2 AU from the Sun. You may assume the dust grain is spherical, and that the albedo of silicate dust is approximately 0.6. [35 pts]

B. Comment whether your computed answer is reasonable or not and explain why. If your answer is unreasonable, assume a more reasonable answer for the two parts below. [5 pts]

C. At what wavelength will the dust emission peak? [20 pts]

D. Identify a telescope that would be able to make observations successfully at this wavelength and explain why this facility would be appropriate. What environmental, telescope, and instrumental conditions are necessary for observations at this wavelength? [40 pts]

QUESTION 7. The Black Hole in the Galactic Center

A. Discuss two lines of evidence that support the conclusion that the nucleus of the Milky Way Galaxy contains a supermassive central black hole. [20 pts]

B. The mass of the central black hole in the Milky Way Galaxy has been determined to be $M_{BH} = 4.0 \times 10^6 M_\odot$. The stellar mass density distribution surrounding the black bole may be approximated by $\rho_*(r) = \rho_0(r/r_0)^{-\alpha}$, with $\alpha = 1.8$. Show that the total stellar mass within r_0 is finite and determine ρ_0 for $r_0 = 1$ pc and $M(r_0) = 1.4 \times 10^6 M_\odot$. How does ρ_0 compare with the stellar density in the solar neighborhood? [40 pts]

C. Show that the black hole increasingly dominates the rotation curve $v_c(r)$ for decreasing $r < r_0$. Use centrifugal force balance to separately compute the contributions of the stars and the black hole to the rotation velocity curve. Sketch each contribution to the rotation velocity curve $v_c(r)$, on $\log v_c - \log r$ axes, and sketch the combined rotation velocity curve, on the same axes. Indicate the location of r_0. [40 pts]

QUESTION 8. Galactic Dynamics

A. Calculate the radial profiles of the density, the circular speed, and the escape speed for a stellar system for which the gravitational potential is equal to

$$\phi(r) = -G\frac{4\pi}{3}\rho_0 a^3/(a\sqrt{1+\frac{r^2}{a^2}}).$$

i. The potential energy of this stellar system is equal to $-\frac{3\pi}{32}\left(G\frac{4\pi}{3}a^3\rho_0\right)^2/a$; what is its total kinetic energy and total energy if the system is in equilibrium?

ii. This system has infinite extension; does it also have infinite mass?

[40 pts]

B. Define the 'final parsec problem' and discuss at least one possible solution proposed in the literature. [30 pts]

C. Describe the process of dynamical friction and its effect on the orbit of a dwarf galaxy or a globular cluster moving inside the Galaxy. [30 pts]

Some of the following expressions and integrals may be useful to answer these questions:

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

$$\int\frac{x^2}{(1+x^2/a^2)^{5/2}}\mathrm{d}x = \frac{1}{3}\frac{x^3}{(1+x^2/a^2)^{3/2}}$$

$$\int\frac{\mathrm{d}x}{(1+x)^2} = -\frac{1}{1+x}$$

QUESTION 9. Galaxies

Consider a hypothetical galaxy in the Virgo Cluster (distance = 16.5 Mpc) with m_B = 10.52 mag, A_B = 0.07 mag, an HI flux density of 18.24 Jy km/s, and a centroid of its HI emission at 1610 km/s.

A. Calculate the distance modulus for this galaxy, assuming it is near the cluster core. Using this value, calculate the absolute B-magnitude of this galaxy (M_B). Show your work. [10 pts]

B. Given the information available, provide a justification for whether this galaxy is likely to be an early-type or late-type galaxy. Include in your answer whether or not you think this may be a dwarf galaxy and your justification for that conclusion. What further observational evidence could you obtain that would help you distinguish between an early-type or late-type galaxy? [15 pts]

C. If you were to detect Hα emission from the center of this galaxy, at what wavelength would it appear? [Recall that the rest-wavelength of Hα is 6563 Å.] Describe the possible sources of Hα emission in this galaxy. Next, describe what additional observations (if any) would be needed to distinguish between different ionizing sources and sketch a diagram (with approximate values on x- and y- axes) that would be relevant to this question. [20 pts]

D. Calculate the peculiar velocity for this galaxy. Show your work and be explicit about any assumptions you have made. [10 pts]

E. Describe one method – other than calculating the galaxy's distance from its redshift – that could potentially be used to find the distance to this particular galaxy. Clearly explain how your chosen method works and how you would go about implementing it. [20 pts]

F. Stellar population modeling of this galaxy indicates two dominant stellar populations, one with a current M_B of -20.23 and B-V color of 0.42 and a second with current M_B of -19.38 and B-V color of 0.86. Based on this information, what is the M_V and B-V color of the galaxy? Show your work. Note that it may be advantageous to calculate M_V for each population before calculating M_V for the galaxy. [25 pts]

QUESTION 10. Big Bang Nucleosynthesis

When the universe has cooled to a temperature T ~ 10^{12} K (time t ~ 10^{-4} sec), the only remaining particles are photons, electron-positron pairs, neutrinos, and a much smaller number of neutrons and protons left over after the annihilation of *all* antineutron and antiprotons with the corresponding particles; this illustrates the universe's tiny excess of particles over antiparticles. The n/p ratio depends on the ambient T and the mass difference between n and p, and at this time is 0.985 (almost unity). Subsequently, n and p are changing into each other rapidly via charged-current weak interactions until the universe cools sufficiently (beginning at T ~ 4×10^{11} K) so that these interactions fall out of equilibrium. (Also, all e^+ annihilate with e^- leaving a tiny number of e^-.) By the time T reaches 10^{10} K (t ~ 1 sec), the n/p ratio has "frozen out" at ~ 1/6.

A. Subsequently, does the n/p ratio continue to evolve, and if so, why? [10 pts]

B. The remaining n and p attempt to synthesize elements, but the nuclear statistical equilibrium (NSE) abundances of deuterium, tritium, and helium-3 (D, T, and ^{3}He) remain extremely small until T ~ 10^9 K (t ~ 3 min), at which point reactions proceed rapidly to establish the NSE abundances of ^{4}He. The result is nearly all n go to ^{4}He, leaving also much smaller (but measurable) abundances of D, ^{3}He, and ^{7}Li. Derive a theoretical expression that relates the resulting mass fraction of 4He to (only) the n/p ratio. [20 pts]

C. Evaluate the numerical value of the ^{4}He mass fraction and explain any assumptions you make. [10 pts]

D. For each of the isotopes, ^{4}He, D, and ^{7}Li, explain (using just 1-3 sentences for each isotope) what kind of observational information is used to try to ascertain their big bang abundances. What issue complicates interpretation of the ^{7}Li observations? [40 pts]

E. Since big bang theory does not specify the (baryon) density of the universe, it is usually treated as a free parameter (often expressed as the baryon-to-photon ratio, η), and the predicted abundances of ^{4}He, ^{3}He, D, and ^{7}Li are plotted as a function of η. If BBN is to be self-consistent, the estimated real abundances of all the isotopes must all imply the same value for η. Explain how the microwave background data (most recently from the *Planck* mission) can be used to infer η. How well does this value of η compare to those inferred from the estimated big bang abundance of the three isotopes listed above? [20 pts]