Ph.D. Qualifying Examination
Department of Astronomy
June 4, 2018
10:00 a.m. — 3:00 p.m.

Name: ________________________

Student Number: _______________

The exam sheets are inside this envelope and are not fastened together. When you are finished, please put the questions and your answer sheets back in the envelope in the correct order. **Be sure the student number given to you by the proctor is on every page of your answers.**

Students pursuing the Astronomy PhD or students pursuing the Astrophysics PhD and taking only the Astronomy qualifying exam **MUST** answer the **FIRST TWO** questions and **SIX** more questions from the remaining **EIGHT**. These students have 5 hours to complete the exam and must finish by **3:00 p.m**.

Students pursuing the Astrophysics PhD who are meeting their qualifying exam requirement by taking part of the Physics qualifying exam and part of the Astronomy qualifying exam must answer **ONE** of the **FIRST TWO** questions and **FOUR** more questions from the remaining **EIGHT**. These students must finish by **1:30 p.m.** (3.5 hours).

M.A. students must do **ONE** of the **FIRST TWO** problems and **THREE** more problems from the remaining **EIGHT**. M.A. students must finish by **12:30 p.m.** (2.5 hours).

Physical Constants

$c = 3.00 \times 10^{10}$ cm/s
$G = 6.67 \times 10^{-8}$ dyn cm^2/g^2
$h = 6.63 \times 10^{-27}$ erg s
$k = 1.38 \times 10^{-16}$ erg/K
$m_p = 1.67 \times 10^{-24}$ g
$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}
$e = 4.80 \times 10^{-10}$ esu
$m_e = 9.11 \times 10^{-28}$ g
$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
$\sigma_T = 6.65 \times 10^{-25}$ cm^2
1 eV $= 1.6 \times 10^{-12}$ erg

$R_\odot = 6.96 \times 10^{10}$ cm
$M_\odot = 1.99 \times 10^{33}$ g
$L_\odot = 3.90 \times 10^{33}$ erg/s
A.U. $= 1.50 \times 10^{13}$ cm
1 year $= 3.16 \times 10^{7}$ s
1 parsec $= 3.09 \times 10^{18}$ cm
$M_{V_\odot} = 4.83$ mag
$B.C._\odot = -0.07$ mag
$(B-V)_\odot = 0.64$ mag
$T_{eff,\odot} = 5770$ K
$M_E = 5.97 \times 10^{27}$ g
$R_E = 6.38 \times 10^{8}$ cm

REQUIRED: QUESTION 1. Beta Centauri A

Beta Centauri A is a spectroscopic binary star comprised of two nearly identical B1 III stars, beta Cen A1 and Beta Cen A2. The beta Cen system includes a third star, beta Cen B, that orbits the beta Cen A binary at some distance with a much longer period.

Data for beta Cen A1 + beta Cen A2

Spectral Type	B1 III + B1 III
Temperature of both A1 and A2	25,000 +/- 2000
Proper motion	-33.27 mas in RA, -23.16 mas in Dec
Parallax	8.32 mas
B magnitude	0.38
V magnitude	0.60
J magnitude	1.167
H magnitude	1.209
K magnitude	1.278
Orbital period	357 days
Semi-major axis (de-projected)	2.8 AU (the stars are not resolved)
Semi-amplitude of the primary	57.4 km s^{-1}
Semi-amplitude of the secondary	72.3 km s^{-1}
Bolometric correction for B1 III stars	-2.6

A. What is the distance (in parsecs) to the beta Cen A binary? [10 pts]

B. Calculate the absolute V magnitudes of beta Cen A1 and beta Cen A2, assuming they are of equal visual magnitude. [25 pts]

C. What is the luminosity of each star in solar luminosities? [20 pts]

D. What is the total mass of beta Cen A1 + beta Cen A2 in solar masses? [25 pts]

E. What are the individual masses of beta Cen A1 and beta Cen A2 in solar masses? [20 pts]

REQUIRED: QUESTION 2. Stellar Evolution

A. On a single HR diagram (with axes of effective temperature and luminosity), sketch out the evolutionary tracks for a 1 solar mass and a 25 solar mass star (assuming solar metallicity) from the Zero Age Main Sequence to its final state. For the 1 solar mass track, label the primary evolutionary stages. [30 pts]

B. Label the axes with appropriate numerical values, indicating the present position of the Sun. [15 pts]

C. Draw on the diagram lines of constant radius at $0.01R_\odot$, $1\ R_\odot$, and $100\ R_\odot$. Give the relationship that describes these lines. [20 pts]

D. Identify and describe 4 key differences in the structure and/or evolution of the 1 solar mass and 25 solar mass star. [25 pts]

E. In a separate diagram, sketch the general features of an observed color-magnitude diagram for a 3 Gyr old star cluster. Label the axes appropriately. Approximately what mass are stars at the main sequence turnoff? [10 pts]

QUESTION 3. The Half-Degree Imager

The WIYN 0.9m consortium employs a CCD camera called the Half Degree Imager (HDI). The HDI has 4096 x 4112 pixels; each pixel is square and 15 microns across. The WIYN 0.9m telescope has a primary mirror diameter of 0.910 m and a focal ratio of f7.478 for the Cassegrain focus (where HDI is mounted). The filters for the HDI camera are square and 4.00 inches on each side.

A. What are the image scale and pixel scale, in arcsec/mm and in arcsec/pixel, respectively? What is the field-of-view (FOV) of the detector, in arc minutes? [25 pts]

B. Compute the theoretical resolving power (a.k.a. Rayleigh criterion) of the telescope in arc seconds at the effective wavelength of the Johnson V-band filter (5500 Å). Based on your knowledge of ground-based imaging, is it likely that this level of angular resolution will be achieved? Why or why not? Elaborate. [25 pts]

C. Calculate the *maximum* height above the CCD that the filter can be placed in the light path without causing vignetting of the final image produced by the filter + CCD combination. [50 pts]

QUESTION 4. Cosmology

Einstein famously referred to the introduction of the cosmological constant (Λ) in 1917 as his "biggest blunder", but today the standard model of cosmology (ΛCDM) is built on the premise of cosmological constant. Einstein's shift from cosmological constant and our eventual return to it were both driven by the data. Einstein constructed his static cosmological model before the discovery of the expansion of the universe; actually before it was even known that the universe extends beyond the Milky Way. In this problem you will calculate some properties of Einstein's universe and compare them to the current cosmology.

A. Use acceleration equation given below (with pressure appropriate for matter-only universe) to show that the cosmological constant in Einstein's universe must be related to matter density. [15 pts]

B. Calculate the matter density that Einstein considered in his model. Start with a rough estimate of MW mass in stars (10^{11} solar masses) and assume it is a simple disk (no bulge) 1 kpc thick. Express your final answer in g/cm^3, rounded to order of magnitude. [15 pts]

C. Derive from Friedmann equation given below the expression for critical density (this assumes no Λ). [15 pts]

D. Calculate the actual matter density (baryonic + dark matter) of the universe in g/cm^3. Use concordance cosmology values of $\Omega_m = \rho_m / \rho_{crit} = 0.3$ and $H_0 = 70$ km/s/Mpc. Why is it so different from Einstein's value from (B)? [15 pts]

E. Show, using Friedmann equation and answer to (A), that Einstein's static universe must be positively curved (closed). [10 pts]

F. Calculate the radius of curvature of Einstein's universe in Mpc. How do you think Einstein regarded such distance scale? [15 pts]

G. An appropriate scale for the actual observable universe is the Hubble distance (ct_H). Calculate it and compare with the scale (radius of curvature) of Einstein's universe. How does the geometry of actual universe differ from Einstein's? [15 pts]

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -4\pi G\left(\frac{\rho}{3} + \frac{p}{c^2}\right) + \frac{\Lambda}{3}$$

QUESTION 5. Earth to Mars

A. The minimum energy orbit that will take a spacecraft from Earth to Mars has its perihelion at Earth's orbit and its aphelion at Mars' orbit. Assuming that Earth and Mars have perfectly circular orbits, and given that the semimajor axis of Mars' orbit is 1.52 AU, what is the semimajor axis, a, and eccentricity, e, of the spacecraft's orbit? How long does the spacecraft take to reach Mars? Sketch the orbits of Earth, Mars, and the space craft. (Recall that the distance from the center of an ellipse to either focus is ae.) [50 pts]

B. Calculate an expression for the energy of a spacecraft of mass m in circular orbit of radius a about the Sun, in terms of the solar mass $M_\odot$, m, a, and physical constants. Given that the energy of an elliptical orbit is determined by its semimajor axis, independent of eccentricity, use this result to derive the vis-viva equation: $v^2 = GM_\odot(2/r - 1/a)$. Finally, determine the speed and direction with which the spacecraft must be launched relative to Earth in order to reach Mars (Neglect the gravitational force of Earth on the spacecraft). [50 pts]

QUESTION 6. Dynamics

A. Galaxy kinematics and morphology [30 pts]

   i. What kinematical properties can be responsible for the ellipticity of elliptical galaxies?
   ii. What plot relating a galaxy kinematics to its morphology has been often used in the literature to address this question? Provide both a sketch and words describing the plot.

B. A stellar system is characterized by a ratio of kinetic, K, to potential energy, W, equal to $K/|W| = 0.1$ [35 pts]

   i. Explain why this system is not in equilibrium, including a brief discussion of the relevant theorem.
   ii. Describe the evolution of this stellar system, and the radial variation of the anisotropy in its velocity distribution emerging after a few dynamical times of evolution as the system evolves toward equilibrium.
   iii. What kind of observational measurement would you need to be able to study the anisotropy profile of this system? (e.g., line-of-sight velocities, proper motions).

C. Two-body relaxation [35 pts]

   i. Describe the fundamental dynamics of two-body relaxation and its timescale.
   ii. Discuss the effects (if any) of two-body relaxation on the evolution of structure and the stellar content of globular clusters.
   iii. Discuss the effects (if any) of two-body relaxation on the evolution of structure and the stellar content of elliptical galaxies.

QUESTION 7. Gaseous Nebulae

A. Define and describe the **nebular approximation** as it relates to ionized nebulae. How does this approximation shape how we model / interpret HII regions? Be sure to include a justification for why the approximation is valid. [20 pts]

B. Define and describe the **on-the-spot approximation** as it relates to ionized nebulae. How does this approximation shape how we model / interpret HII regions? Be sure to include a justification for why the approximation is valid. [20 pts]

C. Consider a pure hydrogen nebula and assume ionization equilibrium applies. Derive a relation for the ionization fraction $f = n(H^+)/n_{tot}$. Be sure to define all symbols used. [30 pts]

D. Assume that the radiation field from the ionizing star is given by

$$J_\nu = 10^{-18} [\nu_o / \nu]^4 \ \ [\text{erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \text{ ster}^{-1}],$$

where $h\nu_o = 13.6$ eV, that the photo-ionization cross section is given by

$$a_\nu = 10^{-17} [\nu_o / \nu]^3 \ \ [\text{cm}^2],$$

and that the relevant recombination rate is

$$\alpha_B = 2.6 \text{ x } 10^{-13} [10^4 / T_e]^{1/2} \ \ [\text{cm}^3 \text{ s}^{-1}] .$$

Adopting these first-order parameterizations for the various quantities relevant to your answer to (C), compute the ionization fraction for the following case: $T_e = 8000$ K and $n_e = 200$ cm^{-3}. [30 pts]

QUESTION 8. Supernova Energetics

A. Suppose that the iron core of an evolved 10 $M_{\odot}$ star reaches the Chandrasekhar mass and collapses to become a neutron star. Estimate the amount of gravitational energy that is released in erg. Discuss your assumptions. [35 pts]

B. Suppose that the envelope of the star is ejected with a velocity of 10^4 km/s. Compute the amount of kinetic energy that is carried away in the escaping envelope material. [20 pts]

C. If the resulting Type II supernova has an optical luminosity of 10^{10} $L_{\odot}$ for a characteristic time of one month, what amount of energy is radiated? [20 pts]

D. Compare your results from parts (A), (B), and (C). What accounts for any remainder of the released gravitational energy? Cite any observational evidence supporting your conclusion. [25 pts]

QUESTION 9. Galaxy Properties

Consider an Sc spiral galaxy that possesses physical properties that are typical of its morphological type. The galaxy appears face-on (i=0), has a B-band central surface brightness of μ_0 = 21.0 mag/sq. arcsec, and is located at a distance of 31 Mpc. The galaxy's surface brightness profile is well fitted by an exponential law with a scale length r_d of 20 arcsec. Recall that the total flux of an exponential disk can be written $F_{Tot} = 2\pi r_d^2 I_0$, where r_d is the scale length and I_0 is the central surface brightness in units of flux/area.

A. Calculate the total apparent B-band magnitude of the galaxy, m_B. [20 pts]

B. If the foreground extinction is A_B = 0.272, calculate (i) the total absolute B-band magnitude, M_B, of the galaxy as well as (ii) the total luminosity of the galaxy in solar luminosities. Recall that M_B = 5.48 for the Sun. [20 pts]

C. Sketch a diagram of what you might expect the HI profile of the galaxy to look like if the galaxy were observed with a single dish radio telescope. In other words, draw a diagram of flux versus heliocentric velocity that shows the approximate shape of the 21-cm detection. Don't worry about the flux units or scale, but do show the approximate location of the velocity centroid in km/s and the expected line-width. Assume a Hubble constant H_0 = 70 km/s/Mpc and that the galaxy has a negligible peculiar velocity. [15 pts]

D. Would you expect this galaxy to have a higher or lower HI-to-total mass (M_{HI}/M_T) than that of an Sa galaxy, such as M65? Explain your answer. [15 pts]

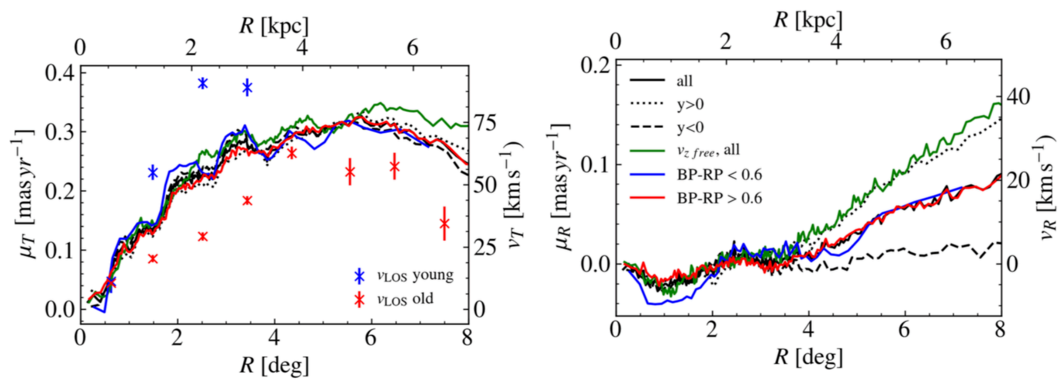
E. One approach to measuring the current star formation rate for galaxies is to use narrowband filters to observe the H-alpha line emission. Given your answer to part (C) above, and the knowledge that rest-frame H-alpha emission is centered at approximately 6563 Å, calculate the expected wavelength at which you would detect H-alpha emission from this galaxy. [10 pts]

F. Sketch the expected surface brightness profiles for both an Sa and an Sc galaxy located at a distance of 31 Mpc. Include representative values on both the x- and y-axes [radius (arcsec) and surface brightness (mag/sq. arcsec), respectively]. Does the central surface brightness of a galaxy depend on distance? Use the definition of surface brightness and relevant equations to clarify your response. [20 pts]

QUESTION 10. Gaia Data Release 2

Data Release 2 (DR2) from the Gaia mission included kinematic data for stars in our Galaxy, in globular clusters, and in nearby galaxies, such as the Small and Large Magellanic Clouds. The following questions are based on data presented in Gaia Collaboration, A. Helmi et al. 2018, arXiv:1808.09381v1.

A. Starting from basic physics, such as Newton's laws, calculate the total mass within r = 7 kpc based on the de-projected rotation curve of the Large Magellanic Cloud shown in the left-hand panel below. Show your work, indicate clearly your adopted values for the various physical parameters, and state any assumptions that you may need to make for this calculation. [25 pts]



B. Neutral hydrogen observations indicate a total 21-cm flux density of approximately 1.0×10^6 Jy km/s for the LMC. Recall that $M_{HI} = 2.356 \times 10^5 D_{Mpc}^2 \int S_{HI}\, dv$. Given a distance of 50.1 kpc (Freedman et al. 2001), what is the total atomic hydrogen mass of the LMC? State clearly any assumptions you may need to make for this calculation. [10 pts]

C. Given your answers to the above questions and a reported B-band total luminosity of $3.0 \times 10^9\ L_\odot$, what percentage of the mass within a radius of 7 kpc of the LMC is non-baryonic? State clearly any assumptions you may need to make for this calculation. [20 pts]

D. An estimate of the dynamical mass of the Milky Way can be calculated in a similar manner based on satellite orbits. One of the most distant galaxies in the Gaia DR2 is Leo I, at an estimated orbital velocity of 217.3 km/s and a distance of 257.8 kpc. Based on this orbital information, what is the dynamical mass of the Milky Way? State clearly any assumptions you may need to make for this calculation. [10 pts]

(Q.10 continued next page)

Q. 10 – Gaia Data Release 2 (continued)

E. Using the virial theorem, calculate the dynamical mass of the globular cluster NGC 6838 based on the following measured properties (kinematic results: Gaia DR2; photometric results: Harris 2010):

parallax:	0.2252 milli-arcsec
velocity dispersion:	3.96 km/s
half-light radius:	1.67 arcmin
central surface density:	676 $M_\odot/pc^3$
absolute V-magnitude:	-5.61

State clearly any assumptions you need to make for this calculation. [15 pts]

F. Briefly discuss the results of the above calculations in the context of the major constituents of the Universe (be sure to include a summary of the expected percentages of the major constituents for the Universe as part of your response). Include in your discussion whether the results above are what you expect for these different classes of objects, and if not, what could account for the discrepancies. For the purpose of this discussion, assume that the luminous component of the Milky Way consists of approximately 1×10^{11} $M_\odot$ of stars and 4×10^{9} $M_\odot$ of gas and dust. Also assume that the luminous stellar component of NGC 6838 is on the order of 1.5×10^{4} $M_\odot$. Show explicitly any calculations that you make and state clearly any assumptions that go into your answer. [20 pts]