Ph.D. Qualifying Examination
Department of Astronomy
May 22nd 2017
10:00 a.m. — 3:00 p.m.

Name: __________________________

Student Number: _________________

The exam sheets are inside this envelope and are not fastened together. When you are finished, please put the questions and your answer sheets back in the envelope in the correct order. **Be sure the student number given to you by the proctor is on every page of your answers.**

Students pursuing the Astronomy PhD or students pursuing the Astrophysics PhD and taking only the Astronomy qualifying exam **MUST** answer the **FIRST TWO** questions and **SIX** more questions from the remaining **EIGHT**. These students have 5 hours to complete the exam and must finish by **3:00 p.m**.

Students pursuing the Astrophysics PhD who are meeting their qualifying exam requirement by taking part of the Physics qualifying exam and part of the Astronomy qualifying exam must answer **ONE** of the **FIRST TWO** questions and **FOUR** more questions from the remaining **EIGHT**. These students must finish by **1:30 p.m.** (3.5 hours).

M.A. students must do **ONE** of the **FIRST TWO** problems and **THREE** more problems from the remaining **EIGHT**. M.A. students must finish by **12:30 p.m.** (2.5 hours).

Physical Constants

$c = 3.00 \times 10^{10}$ cm/s	$R_\odot = 6.96 \times 10^{10}$ cm
$G = 6.67 \times 10^{-8}$ dyn cm^2/g^2	$M_\odot = 1.99 \times 10^{33}$ g
$h = 6.63 \times 10^{-27}$ erg s	$L_\odot = 3.90 \times 10^{33}$ erg/s
$k = 1.38 \times 10^{-16}$ erg/K	A.U. $= 1.50 \times 10^{13}$ cm
$m_p = 1.67 \times 10^{-24}$ g	1 year $= 3.16 \times 10^{7}$ s
$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}	1 parsec $= 3.09 \times 10^{18}$ cm
$e = 4.80 \times 10^{-10}$ esu	$M_{V\odot} = 4.83$ mag
$m_e = 9.11 \times 10^{-28}$ g	$B.C._\odot = -0.07$ mag
$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}	$(B-V)_\odot = 0.64$ mag
$\sigma_T = 6.65 \times 10^{-25}$ cm^2	$T_{eff,\odot} = 5770$ K
1 eV $= 1.6 \times 10^{-12}$ erg	$M_E = 5.97 \times 10^{27}$ g
	$R_E = 6.38 \times 10^{8}$ cm

1. Stellar Evolution: REQUIRED QUESTION

A. Draw an HR diagram and on it sketch the Zero Age Main Sequence (ZAMS). Then sketch the evolutionary tracks for a 1 solar mass star and a 10 solar mass star (assuming solar metallicity) going from the ZAMS to their final states. (40 pts)

B. Label the axes with appropriate numerical values.
(10 pts)

C. Indicate the position of the present day sun on the diagram.
(5 pts)

D. For each mass, indicate clearly on the HR diagram (with numbers or letters) the major phases of the star's evolution. Then for each stage, describe the primary characteristics that define this stage of stellar evolution, including as appropriate, the main energy sources, notable structural features, the main source of energy transport in different regions of the star, the role of degeneracy, etc.
(45 pts)

2. Basic Stellar Properties: REQUIRED QUESTION

Part I:

Two main sequence stars, A and B, are observed to have the following characteristics:

Star	V (mag)	E(B-V)	B.C. (mag)	D (pc)	T_{eff} (K)
A	**9.5**	**0.50**	**3.2**	**2080**	**28,000**
B	**9.5**	**0.00**	**1.0**	**13.8**	**3,480**

Here V represents the V magnitude, E(B-V) is the reddening in (B-V), D is the distance from the Sun, and B.C. is the bolometric correction.

A. Find the ratio of the radius of star A to that of star B.
   (30 pts)

B. What is the approximate spectral type of each star?
   (5 pts)

Part II:

In an unresolved binary system, the total V magnitude of the system is measured to be 12.324 mag. It is somehow determined that the brighter star has V=12.354, from which we can deduce that the secondary has V=16.235 mag.

Numerous spectra are taken of this binary system and although the spectrum of the secondary is not detected, the radial velocity shifts in the lines of the primary result in the orbital phase diagram shown on the next page (taken from Mathieu et al. 1990, The Astronomical Journal, 100, 1859), where P is the orbital period of the system in days. The radial velocities shown are actually vsini and you may assume the inclination angle is 7 degrees. The spectrum of the primary also suggests that the primary is a dwarf.

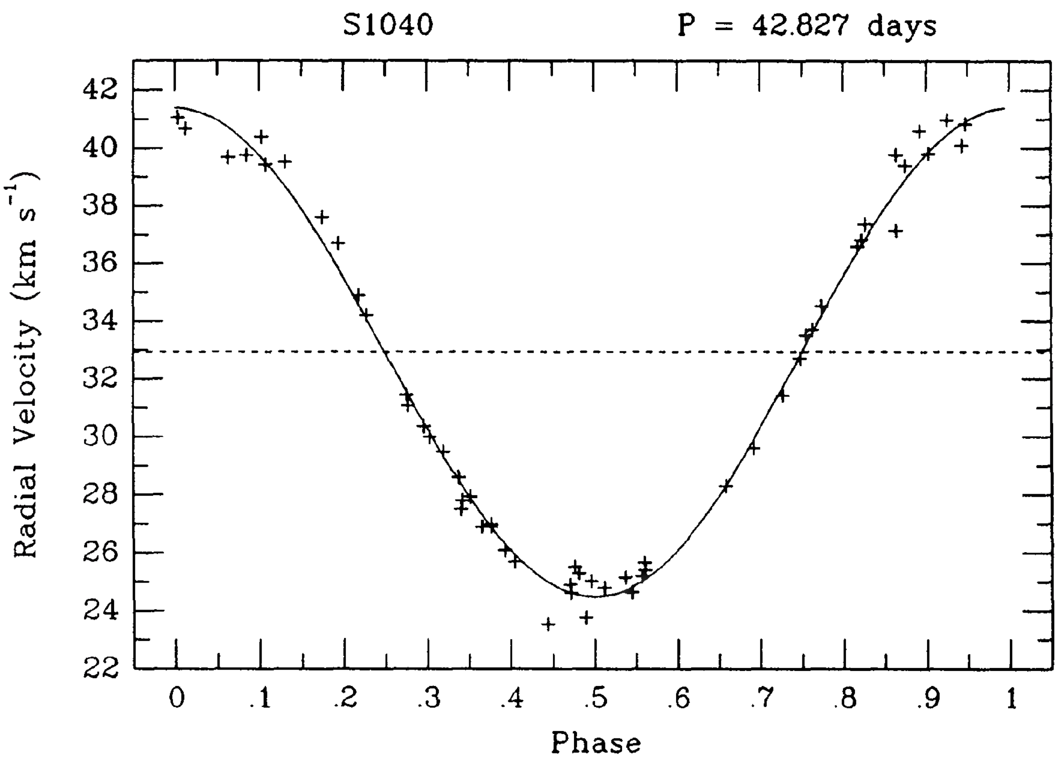
A. What is the approximate shape of the orbit, and why?
   (5 pts)

B. What is the value of the semi-major axis of the orbit, in AU?
   (20 pts)

C. What is the sum of the masses of the two stars, in units of solar masses?
   (15 pts)

D. What are the masses of each of the two stars, in units of solar masses? Note that, to answer this question, you might need to estimate bolometric corrections for the two stars.
   (25 pts)

2. Basic Stellar Properties (cont.)



(from Mathieu et al. 1990, ApJ, 100, 1859)

3. Photometry

A. Sketch the filter bandpasses for the three filters of the Johnson UBV photometric system. Plot relative transmission vs. wavelength. Label each filter and give the approximate central wavelength and FWHM, in Angstroms. The vertical scaling is fairly unimportant, but try to get the wavelength locations and widths approximately correct. (15 points)

B. Draw again the outline of the filters from part (a), but now sketch on top of them the approximate spectral energy distribution for: (i) an O dwarf star, (ii) an A dwarf star, (iii) a G dwarf star, and (iv) an M dwarf star. You don't need to sketch the detailed absorption lines in the stellar spectra; just give the approximate shape of the flux distribution to show where flux of the star lies, relative to the locations of the UBV filters. Label your plot and supplement it with words / short descriptions as appropriate. (25 points)

C. Johnson UBV photometry can be used in a variety of applications to derive important information about stars. Describe in as much detail as you can how UBV photometry is used to measure the following quantities: (i) stellar surface temperature; (ii) metal abundance (define/describe what is meant by "UV excess"); (iii) interstellar reddening (describe/define what is meant by "Color Excess"); (iv) ages of star clusters. The use of diagrams is encouraged. (30 points)

D. Photometry can be a powerful and efficient tool for gaining physical insight into stars, stellar populations, and galaxies. Furthermore, obtaining *accurate* photometry is important to many astronomical applications. Using what you know about astronomical magnitudes and error analysis, show that a 1% error on flux leads to a magnitude error of 0.01 magnitude. (30 points)

4. Black Holes and Active Galactic Nuclei

In April 2017, an international team of astronomers used the Event Horizon Telescope – a combination of existing sub-millimeter facilities, including ALMA – to image the center of the Milky Way in order to detect light from the event horizon of the supermassive black hole believed to be associated with Sgr A*.

A. The observations were conducted at 230 GHz (1.3mm) and used telescopes located in Hawaii, Chile, Antarctica, and Spain. With a longest spatial separation between telescopes of approximately 12000 km, (i) what is the expected spatial resolution (arcsec) of the image created from the Event Horizon Telescope observations? What does this correspond to in terms of a linear resolution (meters) at the distance of (ii) the Galactic Center (d~8 kpc) and (iii) the Virgo Cluster (d~16 Mpc)? (18 pts)

B. Recall that the Schwarzchild radius is given as $r_s = 2GM/c^2$. Based on your answers in (A), calculate the mass limit (in solar masses) for which the Event Horizon Telescope can "see" (resolve) a black hole (i) at the Galactic Center and (ii) in M87 (Virgo Cluster). Clearly indicate if this is an upper or lower limit. (12 pts)

C. In a recent paper, Boehle et al. (2016, ApJ, 830, 17), report an orbital period of 15.92 years for S0-2, which has a semi-major axis of 1.0×10^3 AU, in orbit around Sgr A*. Neglecting relativistic effects, calculate the mass interior to the orbit of S0-2, in units of solar masses. (20 pts)

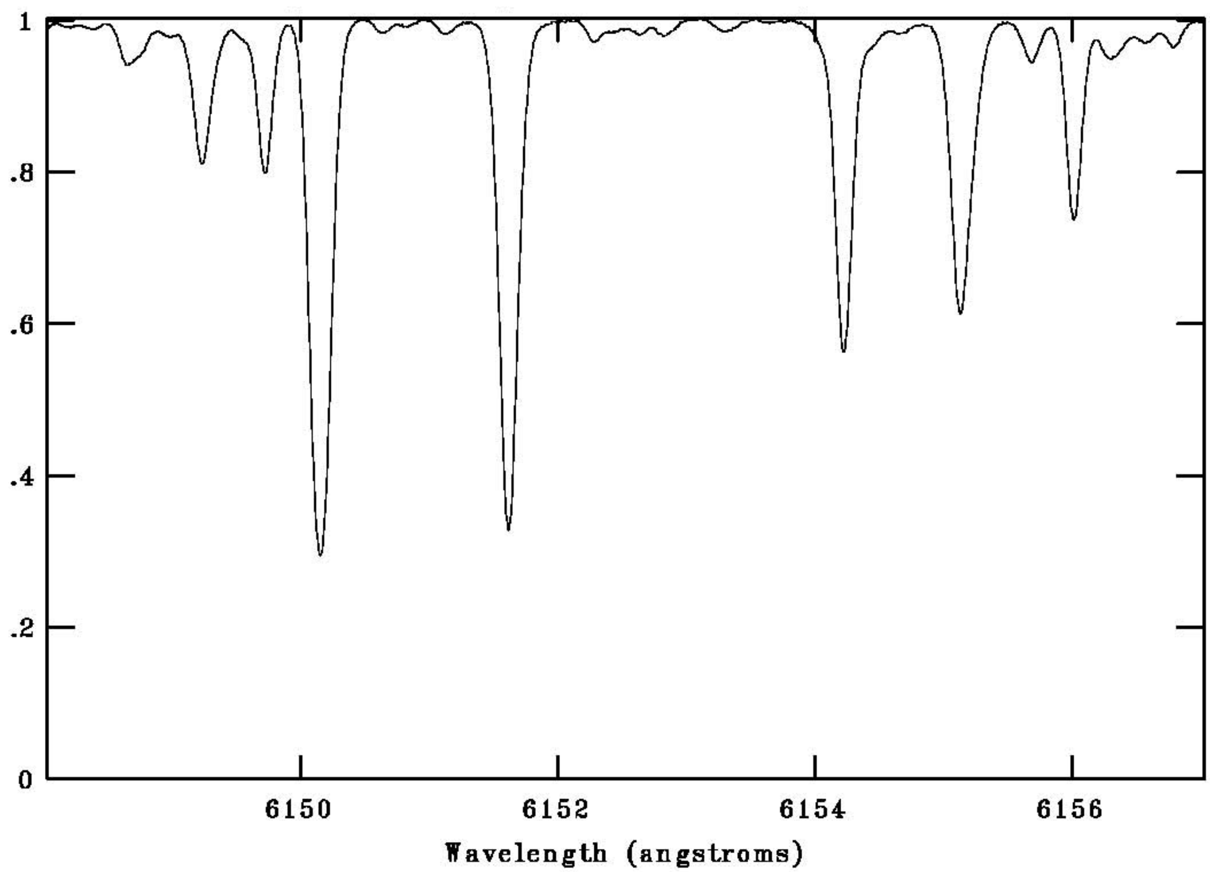
D. Figures in Walsh et al. (2013, ApJ, 770, 86) illustrate a rotation velocity of 623 km/s at a distance of 0.5" from the center of M87. Neglecting relativistic effects, calculate the mass interior to this region, in units of solar masses. (20 pts)

E. Does the Event Horizon Telescope have sufficient spatial resolution to image the black hole at the center of our Galaxy and/or M87? Briefly justify your conclusion based on your answers to (B), (C), and (D). (15 pts)

F. Radio surveys identify numerous sources that may host Active Galactic Nuclei. Consider an AGN located at a distance of 20 Mpc that has an observed bolometric flux of 3.0×10^{-11} erg cm^{-2} s^{-1}, and a T_{eff} of 8×10^2 K. Estimate the angular size of the source in milliarcseconds. Show your work. (15 pts)

5. Spectral Lines

The figure below is a portion of the spectrum of Arcturus, a K3 III giant with a temperature of 4280K, a surface gravity of log g = 1.66, and a metallicity of [Fe/H] = -0.52 according to Ramirez & Allende Prieto (2011, ApJ, 743:135).



Spectral lines of interest in this region include the lines below.

Species	Wavelength	Excitation Potential (eV)
V I	6150.14	0.30
Fe I	6151.62	2.18
Na I	6154.23	2.10
Si I	6155.14	5.62
Ca I	6156.03	2.52

5. Spectral Lines (Cont.)

A. If Arcturus were 200 degrees hotter (but with similar gravity), how would the strength of each of these five spectral features change? Explain your reason for each of the changes.
(25 pts)

B. The strengths of the V I and Fe I features are about the same in Arcturus. How would the relative strengths be different in a star that is 200 degrees cooler but with similar gravity? State your reasoning.
(25 pts)

C. Draw and label a curve of growth. Describe what a curve of growth represents. Define the three general regions of the curve of growth and give the physical reason for the slope of each region in the context of how line strength changes with abundance.
(50 pts)

6. Phases of the ISM

A. Astronomers recognize **six** phases of the interstellar medium (ISM) in the Milky Way. Name each phase, and list a characteristic temperature (in K) and density (#/cm^3) for each. Also give an estimate for the mass fraction of each phase in the Milky Way. You can organize your answer to this section into a table if you wish. (40 points)

B. Explain at least one commonly used method for observing each of the six phases. Your answers for this section should include the wavelengths at which the observations are made as well as the type of telescope used. Be as specific as possible. Appropriate answers should be at least 3-4 sentences in length. (30 points)

C. Some of these phases are in rough pressure equilibrium with each other. Name them, and explain physically what the consequence of being in pressure equilibrium means for these phases of the ISM. (10 points)

D. Likewise, some of the phases of the ISM are **not** in pressure equilibrium. List these, and explain for each whether being out of equilibrium results in an expansion or contraction of the gas. Explain physically what is happening to drive that expansion or contraction in each case. (10 points)

E. Which of the six phases are **directly** associated with the process of star formation. Explain the roles of each phase in the star-formation process. (10 points)

7. Black Hole Tides

A. Consider a star of mass M_S and radius R_S located at a distance d from a black hole of mass M_{BH}. Find an expression for the magnitude of the tidal acceleration at the surface of the star, along the line of centers of the star and the black hole. Assume that $R_S \ll d$ and make an appropriate approximation. (50 pts)

B. Suppose that the tidal acceleration is equal to the gravitational acceleration at the star's surface due to its self-gravity. Use this condition to find the radius r_T of the tidal-breakup sphere around the black hole for stars of radius M_S and radius R_S. (25 pts)

C. Assuming that the supermassive black hole at the center of the Milky Way Galaxy has a mass of $4 \times 10^6\ M_\odot$, determine if a solar type star ($M_S = M_\odot$ and $R_S = R_\odot$) would reach the Schwarzschild radius of the black hole before it were tidally disrupted. (25 pts)

8. Surface Brightness and Stellar Populations

A. Sketch the surface brightness profiles for a disk galaxy and an elliptical galaxy in the B-band. Be sure your sketch includes appropriate values on the x- and y- axes and follows conventions for the orientation of those axes. Beneath your sketches, write the functional forms of these profiles. (20 pts)

B. Show that, to first order, the surface brightness of a galaxy is independent of distance. (15 pts)

C. Freeman (1970) found that most high surface brightness galaxies have a central surface brightness of $\mu_B \sim 21.7$ mag arcsec^{-2}. What quantity does this correspond to in units of $L_\odot$ pc^{-2}? Show your work. (15 pts)

D. In a few sentences, describe both the optical colors and morphologies of late-type and early-type galaxies. (30 pts)

E. In a few sentences, describe the astronomical concepts underpinning the phrase "age-metallicity degeneracy" and discuss what observations might be used to break this degeneracy. (20 pts)

9. Galactic Dynamics

A. Briefly discuss the differences between a collisionless and a collisional stellar system (e.g., physical processes, timescales, numerical techniques). (30 points)

B. Assume that the disk of the Milky Way can be approximated as 2-dimensional, infinitely thin, disk.

(i) Assuming the disk's matter provides the only contribution to the gravity perpendicular to the disk plane, use Gauss's law to derive an expression for the gravitational force normal to the disk plane of a 2D disk with surface density Σ. (15 points)

(ii) A star whose velocity is 30 km/s perpendicular to the plane of the Galactic disk as it crosses the plane is observed to have a maximum departure above the plane of 500 pc. In the thin disk approximation, explain how to estimate the local surface density Σ (do not carry out the calculation). (10 points)

(iii) Different stellar sub-populations of the Milky Way's stellar disk are known to have different vertical scale heights, their thickness increasing with age. Briefly explain the dynamical process(es) thought to underly this correlation and how they would affect part (ii) of this question. (10 points)

C. The surface brightness distribution $I(r)$ of many *elliptical* galaxies is well fit by the luminosity density $j_h(r) = j_o[1 + (r^2/a^2)]^{-3/2}$, where j_o and a are constants. Assume that this luminosity density applies precisely to the *elliptical* galaxy NGC570A.

(i) Stating all assumptions, indicate whether this model has a finite mass. Explain how to determine the circular speed as a function of r (do not carry out the calculation). (15 points)

(ii) Assume that the stellar system is composed of N stars, each with mass m, and can be described by an isothermal velocity distribution given by

$$f(\mathbf{v}) = Ae^{-m(v_x^2+v_y^2+v_z^2)/(2kT)}$$

where A is defined such that $\int_0^\infty f(\mathbf{v})d\mathbf{v} = N$. State the relationship between the mean star kinetic energy and kT. Is the thermal relaxation assumption reasonable for NGC570A. If not, present a reasonable example. Explain. (10 points)

(iii) The velocity-dispersion tensor is given by $\sigma_{ij}^2 = \langle v_i v_j\rangle - \langle v_i\rangle\langle v_j\rangle$. Briefly explain the two terms on the RHS of this equation. Explain their relative importance to NGC570A.
. (10 points)

10. Cosmology

Possibly useful equations:

$$\frac{\dot{R}^2}{R^2} + 2\frac{\ddot{R}}{R} + \frac{8\pi G p}{c^2} = -\frac{kc^2}{R^2} + \Lambda$$

$$\frac{\dot{R}^2}{R^2} - \frac{8\pi G\rho}{3} = -\frac{kc^2}{R^2} + \Lambda/3$$

$$H = (\frac{\dot{R}}{R})$$

$$\frac{R_0}{R} = 1 + z$$

$$H^2(R) = H_0^2[\Omega_\Lambda^0 + \Omega_m^0(R_0/R)^3 + \Omega_r^0(R_0/R)^4 + (1-\Omega_T^0)(R_0/R)^2]$$

1. Friedmann's Equation

A. In a universe that is expanding adiabatically (i.e, the Stefan-Boltzmann law holds), show that

$$(T/T_0) = (R_0/R),$$

where T is the temperature of the CMBR, R is the "scale factor" of the Universe, and "0" = "today".
(15 pts)

B. Use this expression to derive an equation for the variation of temperature as a function of time.
(10 pts)

C. Solve the equation for the variation of temperature as a function of time in the early universe when radiation dominates the energy density. Assume $\Lambda = 0$.
(20 pts)

D. Solve the equation for the variation of the temperature as a function of time if the cosmological constant dominates the energy density. Assume $\Lambda > 0$.
(20 pts)

2. The CMBR photons we see do not originate at the Big Bang but rather "break out" 380,000 yrs afterwards. What prevents astronomers from directly seeing photons that originate earlier than 380,000 years after the Big Bang? What physical process occurs that triggers this "breakout"? What is the naive expectation for the temperature of the CMBR when the breakout occurs? What is the actual temperature of the Universe when the breakout occurs? (35 pts)