Ph.D. Qualifying Examination
Department of Astronomy
June 1st, 2016
10:00 a.m. — 3:00 p.m.

Name: ________________________

Student Number: ________________

The exam sheets are inside this envelope and are not fastened together. When you are finished, please put the questions and your answer sheets back in the envelope in the correct order. **Be sure the student number given to you by the proctor AND the question number is on every page of your answers.**

Astronomy Program students **MUST** do the **FIRST TWO** problems and **SIX** more problems from the remaining **EIGHT**.

Astrophysics Program students **MUST** do **ONE** of the **FIRST TWO** problems and **FOUR** more problems from the remaining **EIGHT**. Astrophysics students must finish by **1:30 p.m.** (3.5 hours).

M.A. students must do **ONE** of the **FIRST TWO** problems and **THREE** more problems from the remaining **EIGHT**. M.A. students must finish by **12:30 p.m.** (2.5 hours).

Physical and Astronomical Constants

$c = 3.00 \times 10^{10}$ cm/s

$G = 6.67 \times 10^{-8}$ dyn cm^2/g^2

$h = 6.63 \times 10^{-27}$ erg s

$k = 1.38 \times 10^{-16}$ erg/K

$m_p = 1.67 \times 10^{-24}$ g

$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}

$e = 4.80 \times 10^{-10}$ esu

$m_e = 9.11 \times 10^{-28}$ g

$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}

$\sigma_T = 6.65 \times 10^{-25}$ cm^2

1 eV = 1.6×10^{-12} erg

$R_\odot = 6.96 \times 10^{10}$ cm

$M_\odot = 1.99 \times 10^{33}$ g

$L_\odot = 3.90 \times 10^{33}$ erg/s

$M_{V\odot} = 4.83$ mag

$B.C._\odot = -0.07$ mag

$(B-V)_\odot = 0.65$ mag

$T_{eff,\odot} = 5770$ K

$M_E = 5.97 \times 10^{27}$ g

$R_E = 6.38 \times 10^8$ cm

1 year = 3.16×10^7 sec

1 A.U. = 1.50×10^{13} cm

1 parsec = 3.09×10^{18} cm

Question 1: The Cosmic Distance Ladder

(a) **Parallax Distance**: Four stars in a hypothetical open star cluster (Cluster "A") are measured to have the following parallaxes: 0.00681, 0.00704, 0.00673, and 0.00692 arcsec. Use these to calculate the average cluster distance. [9 pts]

(b) **The Sun's Properties**: A color-magnitude diagram is attached for stars in the direction of Cluster A, which shows the observed V magnitudes and B – V colors for stars in that direction. (The gridlines are there to help you make needed measurements.) It has been determined independently that there is no measurable interstellar extinction between this cluster and us. Cluster A has the same composition as the Sun. Double-check the distance derived in part a, by using the Sun's properties. You may assume that, for the Sun, B – V = 0.65. [15 pts]

(c) **MS Fitting Distance**: A transparency is provided showing the color-magnitude diagram (the observed V and B – V) for Cluster B, which is also known to have solar composition. However, the interstellar extinction for this cluster is far from negligible: an A0V star in the cluster is observed to have B – V = 0.73. Use main sequence fitting (and the results from above) to determine the distance to this cluster. Which cluster is older, and why? [25 pts]

(d) **Period-Luminosity Relation**: Cluster B is found to have two Cepheids (not shown on the CMD). For the one Cepheid, the observed period is 15.0 days and the observed V magnitude is 7.81, and for the other Cepheid, we have P = 50.0 d and V = 6.34 mag. Using these Cepheids and the known distance from part c, determine the period-luminosity relation. I.e., solve for a and b in the relation MV = a (log P) + b . [15 pts]

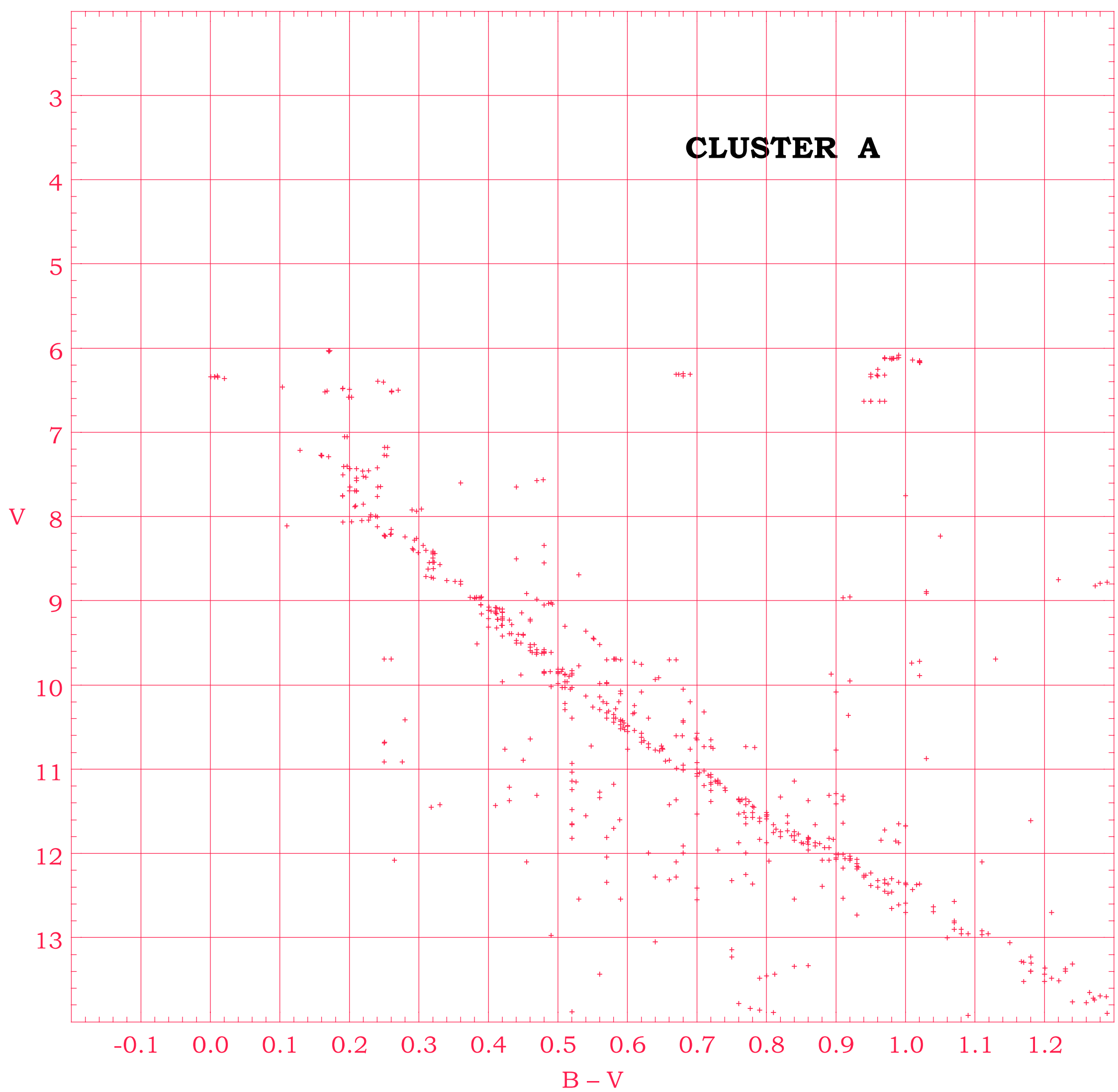
(e) **Cepheid Distance**: Galaxy C is observed to have a Cepheid with period 31.56 days. For this Cepheid, V = 23.37, and there is no reddening in the direction of the galaxy. Using part d, find the distance to this galaxy. [12 pts]

(f) **Hubble Constant**: Spectra of galaxy C reveal it has a recession velocity of 529 km/s. (This is due purely to the expansion of the Universe; i.e. all local gravitational effects have been taken out.) Use this, the distance to the galaxy derived in part e, and the Hubble law to determine the Hubble constant. [12 pts]

(g) **Redshift Distance:** Observations of a distant super cluster of galaxies show the H⍰lpha line centered at 7215 Å. Assuming the value for the Hubble constant from part f, and that the Hubble law is still a good approximation, determine the distance to this super cluster. [12 pts]

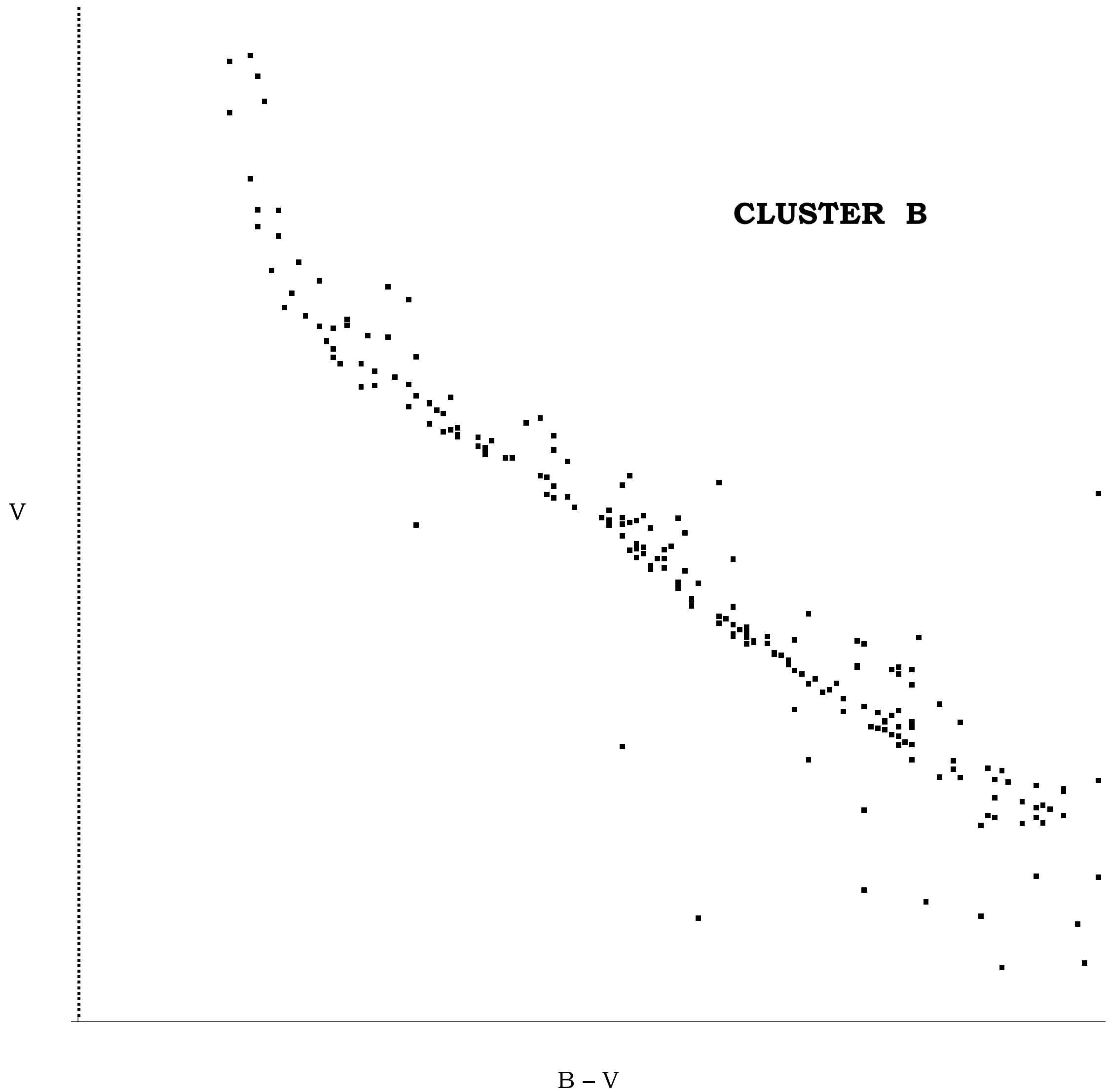
Question 1 (continued)

Figure for parts (b) & (c).



Question 1 (continued)

Figure for parts (b) & (c). Also provided as a transparency for comparison with Cluster A.



Question 2: A Galaxy Collision

(a) Suppose that the Milky Way and M31 are each described by a flat rotation curve that extends to 100 kpc from the galaxy center with a rotation velocity of 220 km/s. Use this to estimate the mass of each galaxy, expressed in solar masses. Assume that each galaxy has a luminosity of 2×10^{10} $L_{\odot}$. Estimate the overall *M/L* ratio, expressed in solar units, and discuss the implications for the content of the galaxy halos.
[30 pts]

(b) Now suppose that the two galaxies *are just beginning to fall together* from their present separation of 700 kpc. Assume that there is no transverse motion. Estimate the amount of time required for the two galaxies to collide in years. (Approximate each galaxy as a point mass.)
[40 pts]

(c) Discuss the likely result of such a collision between the Milky Way and M31, including both transient effects and the long-term outcome. Estimate the relevant timescale for the establishment of the long-term outcome.
[30 pts]

Question 3: The High Redshift Universe

Current searches for high-redshift objects include identifying " –drop-outs" in Ultra Deep fields. The highest-redshift object reported so far this year (2016) is GN-z11, identified in the GOODS North field as part of the CANDELS survey. The WFC3/IR grism spectra of GN-z11 indicates a significant continuum break at $\lambda = 1.47\ \mu m$ (Oesch et al. 2016, ApJ, 819, 129). A press release states that GN-z11 "is seen as it was 13.4 billion years in the past, just 400 million years after the big bang." However, there are actually 3 highly probable interpretations of the continuum break observed in GN-z11:

i. The continuum break corresponds to the Lyman break (121.6 nm)
ii. The continuum break corresponds to the 4000 Angstrom break
iii. The "continuum break" is actually an extremely strong emission line, such as [O III] 500.7nm, appearing at $1.47\ \mu m$.

The following questions ask you to explore the nature of GN-z11 based on these three possible interpretations.

a. What "type" of drop-out is GN-z11 (i.e., the continuum break falls within which filter)? [5 pts]

b. Calculate the precise redshift of GN-z11 for each of the three possibilities listed above. [30 pts]

c. If (i) is correct, what is the apparent recessional velocity of GN-z11, with respect to the speed of light? Recall that $\lambda/\lambda_0 = [(1+v/c)/(1-v/c)]^{½}$. [15 pts]

d. One argument for why GN-z11 is a galaxy, not a QSO, is an apparent lack of variability over a 1 year (observed) time scale. Using a cosmological interpretation of the redshift, and assuming that (i) is correct, what is the corresponding time scale at the source of emission? If GN-z11 did show evidence of variability on this time scale, what would be the maximum size-scale of the source of variable emission (in parsec)? Recall that $\Delta t_{true} = \Delta t\ (1 - v^2/c^2)^{½}$ for relativistic motion. [20 pts]

e. Describe how one might be able to distinguish between (i), (ii), and (iii) (i.e., what observational data would be needed and how would this knowledge be used to break the apparent degeneracy?). Provide specific examples of the expected shape of the redshifted continuum emission for observations with broadband filters and/or the observational wavelengths predicted for redshifted emission and/or absorption lines. Be careful to ensure that your proposed observations are feasible with current or planned instrumentation (ground-based or space-based). [30 pts]

Question 4: Observational Astronomy

Answer the following questions on basic statistics in the context of astronomy and astrophysics. Be clear and complete in your answer, giving quantitative information wherever possible.

(a) Explain the difference between precision and accuracy.
[5 pts]

(b) What are systematic errors? What are random errors? Explain how random errors can be reduced in an experiment or series of experiments.
[10 pts]

(c) What is a Poisson distribution? To what types of experiments/situations does it apply? Draw an example Poisson distribution and label (approximately) the mean, standard deviation, and mode of the distribution. Write an expression for the standard deviation as it relates to the mean value of the Poisson distribution. Finally, provide a specific example from astronomy to which a Poisson distribution would apply; explain your answer.
[30 pts]

(d) What is a Gaussian distribution? To what types of experiments/situations does it apply? Draw an example Gaussian distribution and label (approximately) the mean, standard deviation ("sigma"), mode, median, and full width half maximum (FWHM) of the distribution. Mark and label the locations of the 1-sigma, 2-sigma, and 3-sigma distances from the mean of the distribution, and explain what they mean and why they are important. Finally, provide a specific example from astronomy in which a Gaussian distribution would apply; explain your answer.
[30 pts]

(e) Suppose an astronomer is using a photon-counting detector to measure the flux from a single star. When the detector is pointed at the target star, it collects photons from both the star and immediate area around it (the background sky). The astronomer finds that when the detector is pointed at a patch of blank sky near the target star, the measured background count level is one-fourth of what is measured when the detector is pointed at the star. Given this situation, how many photons should the astronomer collect with the detector (when it is pointed at the star) in order to achieve a relative precision of 2% in the determination of the star's flux? What would be the corresponding magnitude error for this flux measurement? Explain.
[25 pts]

Question 5: Quasars

(a) What are quasars (a.k.a. QSOs)? In approximate numbers, give estimates for the luminosity (in solar units or absolute magnitude) and distance (in Mpc) for a typical quasar.
[20 pts]

(b) What is the current best model to explain the energy output of quasars? Give as many details as possible, including the observational evidence that supports this model.
[30 pts]

(c) Quasars are observed to be more common at higher redshift than they are in the nearby universe. Provide an explanation for why this may be.
[20 pts]

(d) Spectra of distant quasars often show hundreds of weak, narrow absorption lines blue-ward of the Ly-alpha emission line from the quasar. These are referred to as "Lyman-alpha forest lines". What is causing this absorption? What can we learn by studying these absorption lines?
[20 pts]

(e) Why don't we see any Lyman-alpha forest lines red-ward of the Ly-alpha line emitted by the quasar?
[10 pts]

Question 6: Cosmology

1. For Friedmann models with Λ = 0 (Fig. 1 below), show that $1/H_0$ is an upper limit to the age of the universe.
   [20 pts]

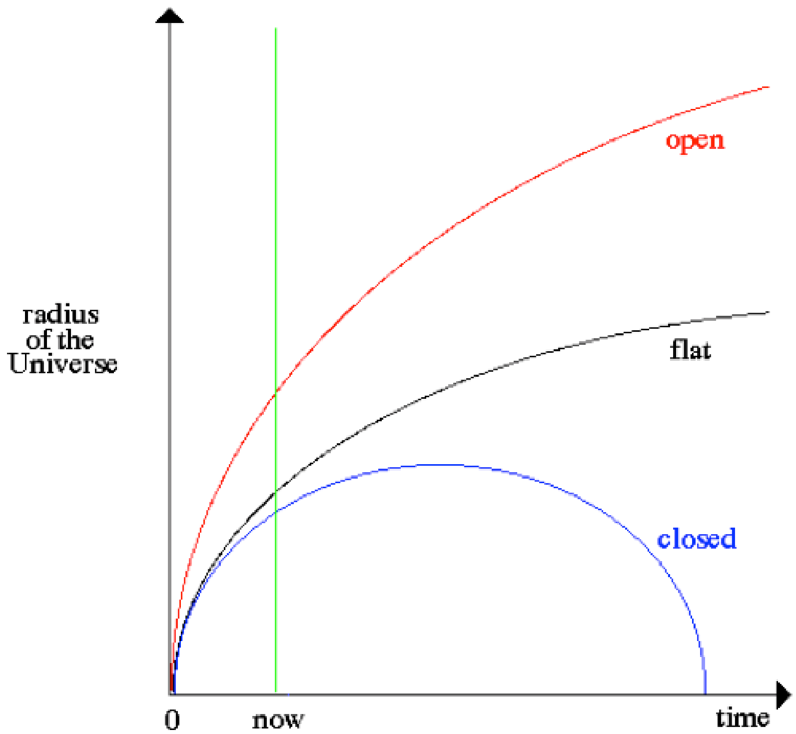


Figure 1: Friedmann models with Λ = 0

2. Big Bang Nucleosynthesis.

   (a) Give a timeline for, and explain the events leading to, the production of the light elements in the Universe, starting with t ≈ 0.1 sec before weak force decoupling (WFd), until t ≈ 3 min, when the light elements are produced. Specifically, what holds up BBN until t ≈ 3 min? [30 pts]

   (b) Show that the baryon to photon ratio $\eta = n_b/n_\gamma$ is the same today as it was at t = 3 min after BBN. [20 pts]

   (c) Figure 2, found on the next page, represents the results of Big Bang nucleosynthesis calculations using the standard ΛCDM model of cosmology. How does this figure demonstrate that most of the matter in the Universe must be different from the "ordinary" matter, like atoms and molecules that we see on Earth? [30 pts]

Question 6 (continued)

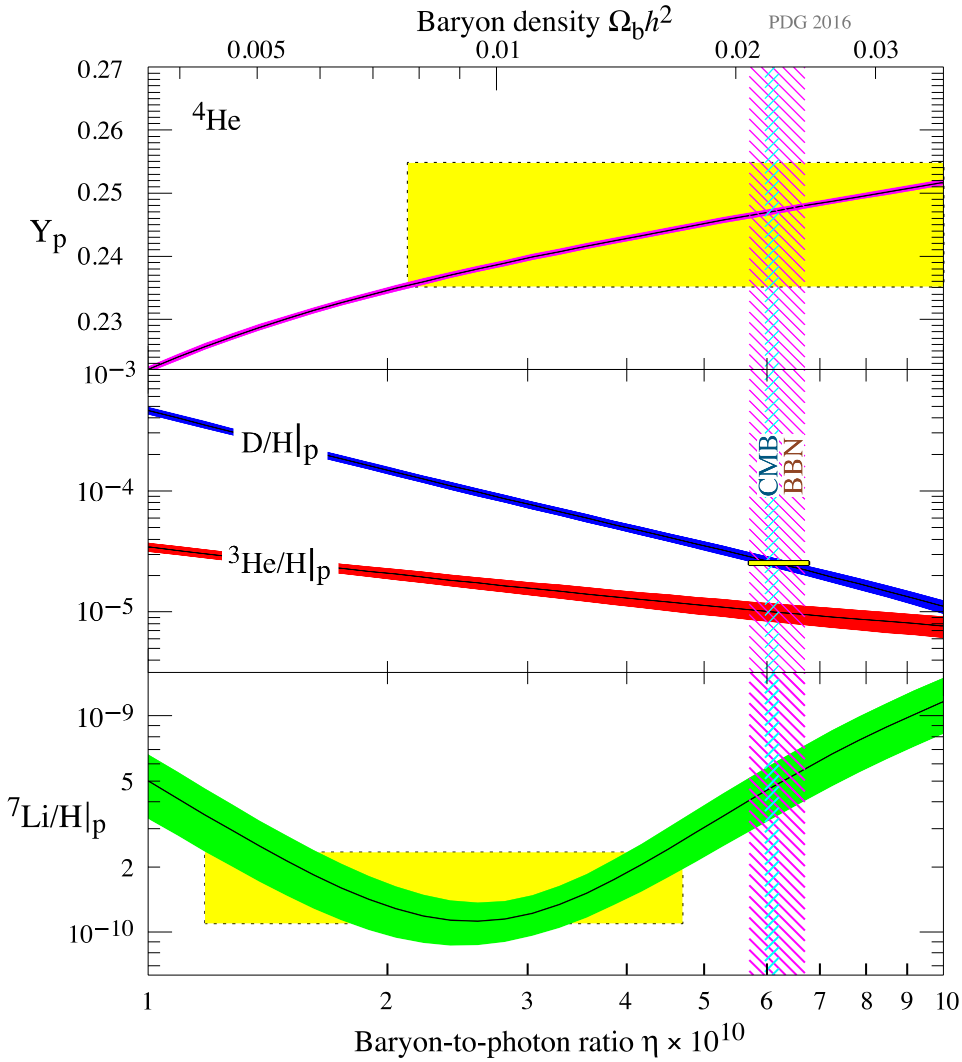


Figure 2: Results of Big Bang nucleosynthesis calculations using the standard ΛCDM model.

Question 7: Virial Theorem Applications

(a) Give a statement of the virial theorem and simple expressions for the potential and kinetic energies of a stellar system of total mass M and characteristic radius R. Explain how the virial theorem can be used to estimate the total masses of stellar systems. What types of observations are needed for this application? For what type of systems is this approach most useful?
[40 pts]

(b) Consider an isolated globular star cluster that is initially in virial equilibrium. Suppose that a fraction f of its mass is instantaneously removed at each radius, as a result of supernova explosions. By separately considering the instantaneous changes in the kinetic and potential energies that result from the mass loss, show that the system becomes unbound if $f > 0.5$.
[40 pts]

(c) Assuming that the globular cluster remains bound, estimate the amount of time in years required for it to reach a new state of virial equilibrium. You may use any reasonable values for the mass and size of the cluster.
[20 pts]

Question 8: Building a Model of the Milky Way

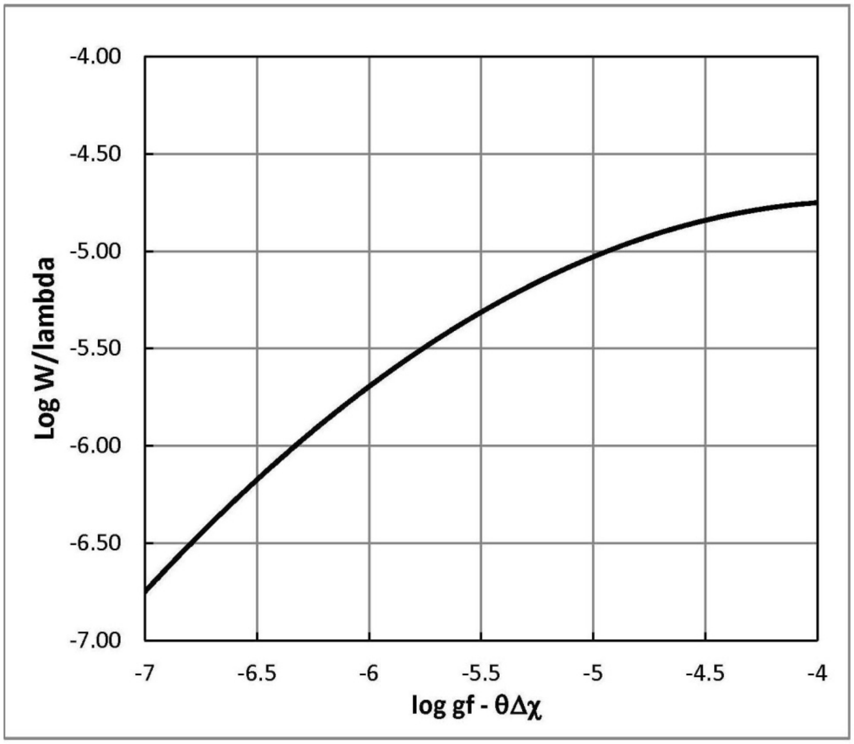
Suppose you are tasked with designing a model that will describe the populations and components of the Milky Way galaxy, like the Besancon or Trilegal star count models, and predict what you would see if you were to 'observe' the Galaxy from the vantage point of Earth. Your model should be capable of predicting the apparent magnitudes and colors of stars you would observe along any line of sight in the Galaxy and to a given apparent magnitude.

a. Outline the main ingredients that you would need to consider incorporating when constructing a model that successfully matches the properties of the Milky Way and its Galactic stellar components. For each of the ingredients or concepts that you identify, describe commonly used assumptions, prescriptions, or functional forms that would serve as input. Be as inclusive as possible in your answers.
[70 pts]

b. Identify 3 aspects of the model where you consider current uncertainties in input to be most important in limiting its reliability or accuracy. Describe how these uncertainties impact the predictions of the model.
[30 pts]

Question 9: Stellar Atmospheres

A curve of growth for Fe I lines in a narrow range of wavelengths near 6000 A in the solar spectrum is shown below. The curve is calculated for lines with an excitation potential of 0 eV, and the abundance of iron in the Sun is A_{Fe} = 7.5 on a scale where A_H = 12.



1. Estimate the abundance that would be derived from a 20 mA Fe I line in the same region of the spectrum if the line has an excitation potential of 4 eV and log gf = -5.0. [40 pts]

2. Would the Fe I line above be weaker or stronger if the temperature of the Sun were 6000 K instead of 5780K? Why? [20 pts]

3. Would the Fe I line above be weaker or stronger if the surface gravity of the Sun were log g = 3 instead of log g = 4.4? Why? [20 pts]

4. In the figure above, draw in how the shape of the curve of growth would differ if the microturbulence in the Sun was 2 km s^{-1} instead of 1 km s^{-1}. [20 pts]

Question 10: Galactic Dynamics

I. Cluster Binaries

(a) Write the expression of the parameter used to classify binary stars in a star cluster as 'hard' or 'soft'.
[10 pts]

(b) What is the role of binary stars in the dynamical evolution of a star cluster?
[10 pts]

(c) Which are the important binaries for the dynamical evolution of a star cluster: the 'soft' or the 'hard' ones?
[10 pts]

(d) What happens to the binding energy of hard and soft binary stars as a result of encounters with single stars?
[10 pts]

(e) Sketch the evolution of the Lagrangian radii in a globular cluster and identify the point when binary interactions become important for the evolution of the cluster structure.
[10 pts]

II. Velocity Anisotropy

(a) What is the value of the anisotropy parameter β [defined as $\beta = 1 - \sigma_T^2/(2\sigma_r^2)$, where σ_T is the total tangential velocity dispersion and σ_r is the radial velocity dispersion] for a stellar system characterized by a distribution function f that depends only on the star energy E. Explain your answer.
[30 pts]

(b) Draw the typical radial profile of β for a stellar system at the end of a 'violent relaxation' simulation. Could this system be described by a distribution function depending only on E?
[20 pts]