Ph.D. Qualifying Examination
Department of Astronomy
May 27th, 2015
10:00 a.m. — 3:00 p.m.

Name: ___________________________

Student Number: _________________

The exam sheets are inside this envelope and are not fastened together. When you are finished, please put the questions and your answer sheets back in the envelope in the correct order. **Be sure the student number given to you by the proctor is on every page of your answers.**

Astronomy Program students **MUST** do the **FIRST TWO** problems and **SIX** more problems from the remaining **EIGHT**.

Astrophysics Program students **MUST** do **ONE** of the **FIRST TWO** problems and **FOUR** more problems from the remaining **EIGHT**. Astrophysics students must finish by **1:30 p.m.** (3.5 hours).

M.A. students must do **ONE** of the **FIRST TWO** problems and **THREE** more problems from the remaining **EIGHT**. M.A. students must finish by **12:30 p.m.** (2.5 hours).

Physical Constants

$c = 3.00 \times 10^{10}$ cm/s
$G = 6.67 \times 10^{-8}$ dyn cm^2/g^2
$h = 6.63 \times 10^{-27}$ erg s
$k = 1.38 \times 10^{-16}$ erg/K
$m_p = 1.67 \times 10^{-24}$ g
$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}
$e = 4.80 \times 10^{-10}$ esu
$m_e = 9.11 \times 10^{-28}$ g
$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
$\sigma_T = 6.65 \times 10^{-25}$ cm^2
1 eV = 1.6×10^{-12} erg

$R_\odot = 6.96 \times 10^{10}$ cm
$M_\odot = 1.99 \times 10^{33}$ g
$L_\odot = 3.90 \times 10^{33}$ erg/s
A.U. = 1.50×10^{13} cm
1 year = 3.16×10^7 s
1 parsec = 3.09×10^{18} cm
$M_{V\odot} = 4.83$ mag
$B.C._\odot = -0.07$ mag
$(B-V)_\odot = 0.64$ mag
$T_{eff,\odot} = 5770$ K
$M_E = 5.97 \times 10^{27}$ g
$R_E = 6.38 \times 10^8$ cm

REQUIRED: Question 1: Stellar Populations

Based on the figures shown on the next page, answer the following questions.

(a) The following BV color-magnitude diagram has been observed in a field in the direction of galactic coordinates $l = 260°$, $b = -22°$. Identify the principal features you see in the CMD, label them clearly on the diagram, and explain what kinds of stars contribute to that feature of the CMD.
[25 pts]

(b) Explain several ways by which you could determine the distance to this object. Based on one or more of these methods, give an estimate of its distance. Show your work and explain any assumptions you make.
[25 pts]

(c) Describe the star formation history of the stellar populations that would have led to this CMD. Be as quantitative as possible in terms of timescales and relative intensity. Explain your reasoning and what evidence in the CMD leads to your conclusions.
[25 pts]

(d) Spectroscopic abundances have been determined for some of the brightest stars in the field and are represented in the figure below by the large filled star symbols; smaller filled circles in the diagram are Milky Way field populations for comparison. What could you deduce about the chemical enrichment history of the object from the distribution shown? Explain your reasoning.
[25 pts]

Question 1 (continued)

Figure for parts (a), (b), and (c).

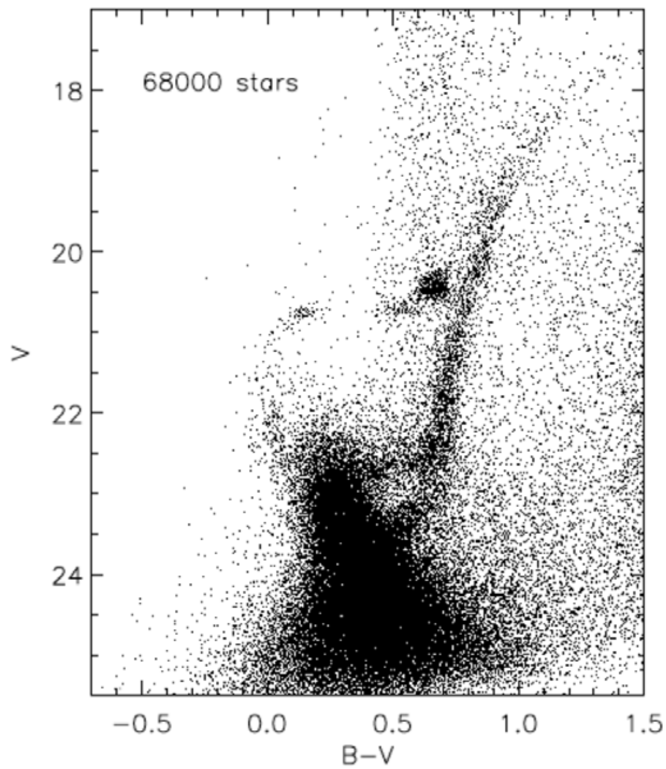
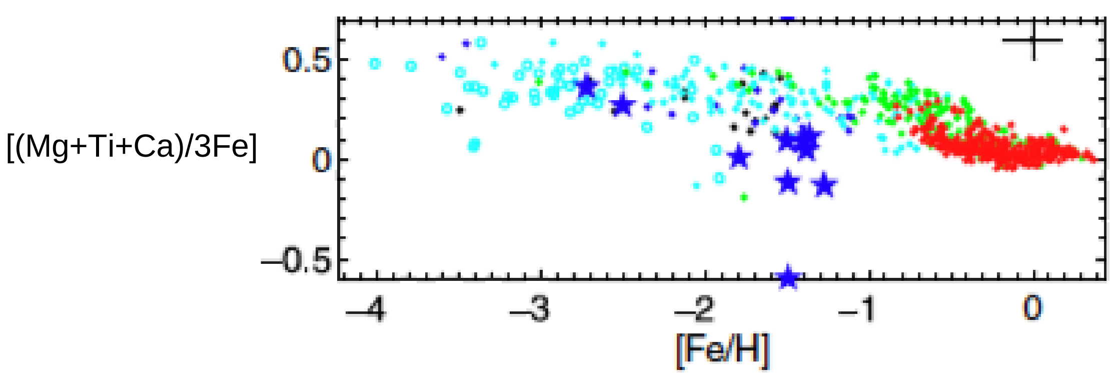


Figure for part (d).



REQUIRED: Question 2: Brown Dwarf Planet Host

A planet has been discovered near the young brown dwarf 2M1207, which itself is a member of the TW Hya association. The star and planet are separated on the sky by 0.8 arc seconds, and the parallax of 2M1207 is 19 milli-arc seconds. 2M1207 has a mass estimated at 0.025 solar masses and the mass of 2M1207b is estimated to be about 0.005 solar masses. The brown dwarf 2M1207 has a temperature near 2500 K and its luminosity is 0.002 solar luminosities.

For reference, sample albedo values include the following.

Dark rock	0.05	Clouds	0.5
water	0.10	Ice	0.6
Bare soil	0.17	Fresh snow	0.9
Grass	0.25		

Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}$ W m^{-2} K^{-4}

(a) What is the distance to 2M1207 in parsecs?
[10 points]

(b) What is the minimum physical separation of the planet from 2M1207 in AU?
[20 points]

(c) If the planet orbits in a circular orbit at the minimum separation, what would its orbital period be?
[20 points]

(d) What would you expect the equilibrium average temperature of 2M1207b to be?
[40 points]

(e) Based on its luminosity and color, the temperature of 2M1207b is measured to be 1600 K. What is the most likely explanation for the difference between the observed and predicted temperature of the planet.
[10 points]

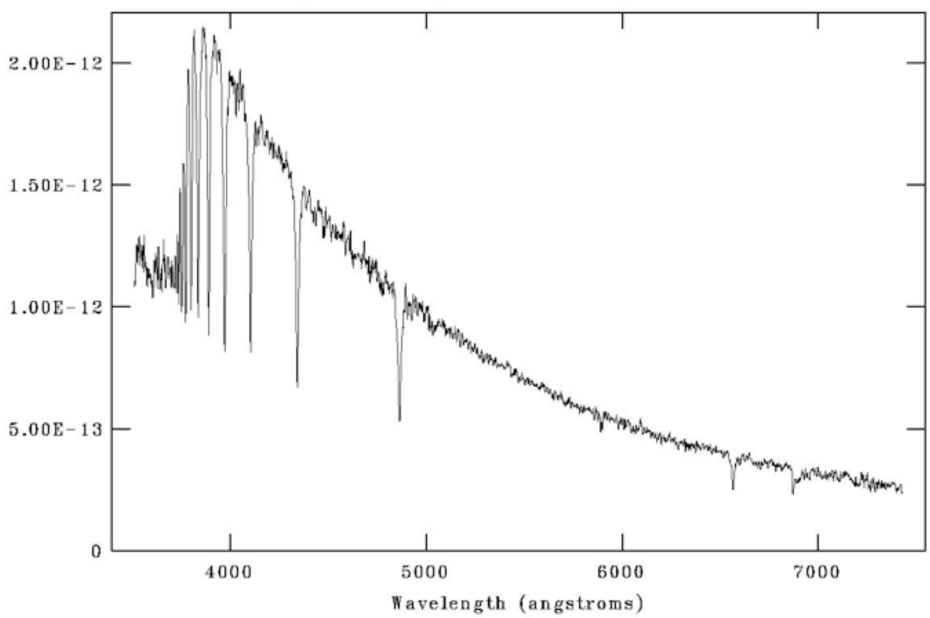
Question 3: Stellar Atmospheres

Three stellar spectra (A, B, & C) are shown below and on the following page. For each of the three spectra:

(a) Estimate the approximate spectral type (luminosity classes are not needed) and temperature, and describe what information you used to estimate the temperature, including both continuum and line features. Label the figure with these features and any others you can identify.
[50 points]

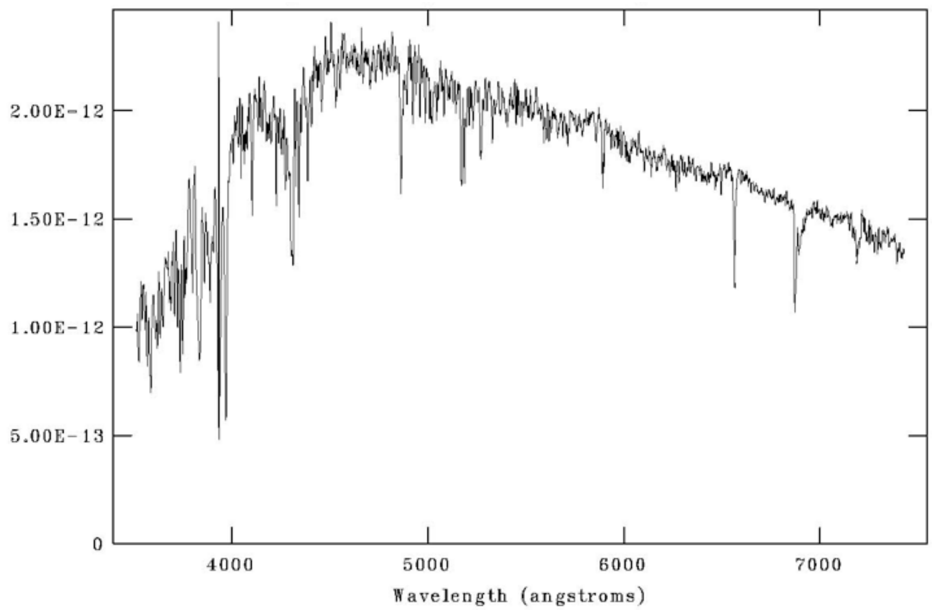
(b) For each star, identify the most important source of opacity in the optical region of the spectrum. Explain why this source dominates over other sources of opacity and indicate how both the temperature and the opacity affect the overall shape of each spectrum.
[50 points]

A.

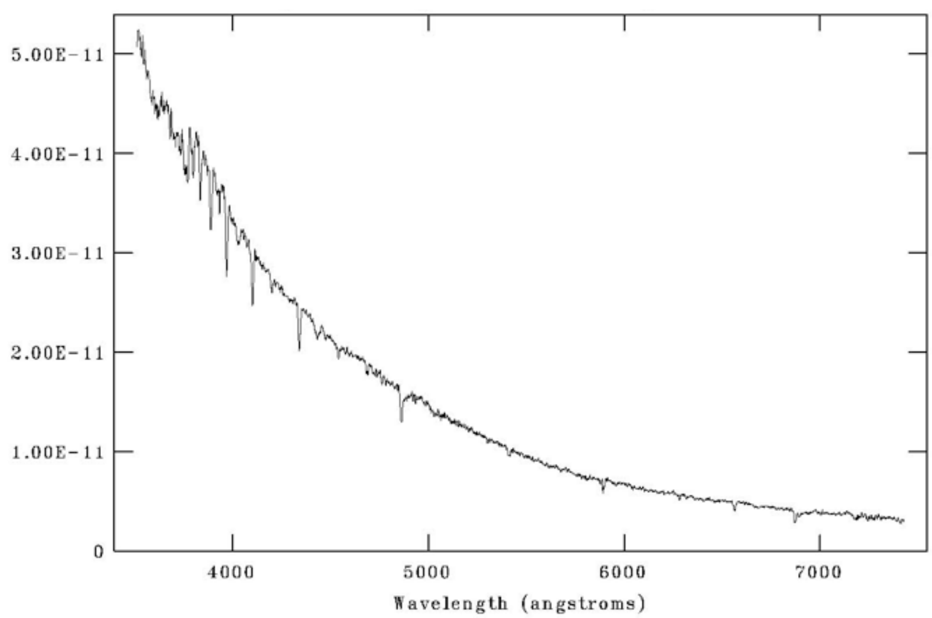


Question 3 (Continued)

B.



C.



Question 4: Galactic Structural Parameters

Consider a face-on galaxy in the Virgo cluster (D=16 Mpc) that can be modeled as a system composed of only 2 stellar populations:
(1) a 13 Gyr old population with M_B = -15.2 and (B-V) = 0.9, and
(2) a 1 Gyr old population with M_B = -15.6 and (B-V)=0.4.

(a) Calculate the integrated **absolute** (M_B and M_V) and **apparent** (m_B and m_V) magnitudes at both B and V-band and the **integrated (B-V) color** of the galaxy. Show your work.
[30 points]

(b) The Sérsic profile is $I(R)= I(R_s) \exp(-R/R_s)^n$. Re-write this equation in terms of surface brightness as a function of radius, $\mu(R)$, rather than intensity as a function of radius. Show your work.
[10 points]

(c) Assuming a single exponential surface brightness profile with a scale length of 1 kpc, what is the central surface brightness in B-band (mag arcsec^{-2}) of this galaxy? What is the angular size corresponding to D_{25}? Based on your knowledge of the selection criteria of typical galaxy catalogs, is this galaxy likely to be included in the NGC or UGC? Is it likely to be a target for SDSS spectroscopy? Justify your answers.
[25 points]

(d) Estimate the total hydrogen mass associated with this galaxy if it is a typical spiral or irregular galaxy. Further, recalling that $M_H = 2.356 \times 10^5 D^2 \int S\, dv$, where D is in Mpc and $\int S\, dv$ is in Jy km s^{-1}, estimate the peak flux density of the 21cm line (in Jy). What signal-to-noise ratio would you have in this line if you observed this galaxy with the VLA in D configuration for 1 hour if the typical noise measurement is 2.0 mJy/beam in a 1 hour integration at L-band? Clearly state any assumptions that you must make for these estimates.
[25 points]

(e) Calculate the dynamical mass of this galaxy if it has a bound companion at a projected distance of 10 kpc and a relative velocity of 180 km s^{-1}. Show your work.
[10 points]

Question 5: The Light From Galaxies

(a) Show that, to first order, the surface brightness of an object (e.g., a galaxy) is independent of distance. Be as rigorous as possible.
[25 points]

(b) You observe an E0 elliptical galaxy that has a total B-band apparent magnitude of 11.85 and an effective radius of 23 arcsec. What is the MEAN surface brightness (in mags/sq. arcsec) inside the effective radius?
[25 points]

(c) Consider two galaxies of equal mass, one elliptical and one spiral (type Sc). In general, such galaxies will have significantly different B-V colors. Characterize (be as quantitative as possible) the colors of the two galaxies. State the primary reason why these galaxies would have different observed colors.
[25 points]

(d) Now consider two elliptical galaxies of vastly different mass – one giant and one dwarf. In terms of appearance and star-formation histories, these two galaxies are similar. In general, however, such galaxies will also have significantly different B-V colors. Characterize (qualitatively at least) the colors of the two galaxies. State the primary reason why these galaxies would have different observed colors.
[25 points]

Question 6: A Galactic Bulge Model of Star Clusters

(a) Consider the following idealized model for a spherically symmetric bulge component of a galaxy:

$$\Phi(r) = -GM_0/(r + r_0),$$

where M_0 and r_0 are constants.

Compute and sketch the behavior of the following radial profiles for this bulge model:

i) the bulge mass contained within radius r, $M(r)$

ii) the rotation curve, $v_c(r)$

iii) the bulge density profile, $\rho(r)$

In each case, compute the exact behavior of the functions and then give asymptotic forms for the cases $r \ll r_0$ and $r \gg r_0$. Make sketches of each radial function on log-log axes. Is there anything unphysical about this idealized bulge model? What other components of the potential are necessary to make a realistic model for a spiral galaxy?

[60 pts]

(b) Suppose that the maximum rotation velocity of the galaxy is observed to be 300 km/s at $r =$ 3 kpc. Use this to compute the total mass of the bulge.

($G = 6.7 \times 10^{-8}$ cm^3 g^{-1} s^{-2}, $M_\odot = 2.0 \times 10^{33}$ g, and 1 pc $= 3.1 \times 10^{18}$ cm)

[40 pts]

Question 7: CCDs

(a) Consider a CCD detector that is attached to a telescope and is used to image the sky at optical to near-infrared wavelengths. Thoroughly answer the following questions about the properties and operation of the CCD in the context of astronomical observations. **Where possible, use quantitative expressions and/or diagrams to help explain your answer.**

(i) What type of solid material is the CCD likely to be made of? What fundamental property does this material have (when compared to other types of solids) that makes it effective for detecting photons via the photoelectric effect?
[10 points]

(ii) Most research-grade CCD detectors are intrinsically sensitive to photons in the range ~400 nm to ~1100 nm. What determines the range of photons that the CCD will be sensitive to? What steps can be taken to increase the sensitivities of CCDs at the short- and long-wavelength ends of the response curve? Explain clearly how each technique or approach works to increase the sensitivity in these regimes.
[30 points]

(iii) Explain in detail how the CCD is operated – how does the device "capture" and "store" the incoming photons? How is the CCD "read out" to form a digital image once the exposure is finished?
[30 points]

(b) What is an orthogonal transfer CCD (OTCCD)? How does it differ from a conventional CCD detector?
[10 points]

(c) Consider a good-quality, low read-noise CCD detector that is operated in a cooled mode and has a negligible dark rate. Write a general expression for the signal-to-noise ratio (S/N) per pixel of an astronomical object (in terms of count rate(s) and exposure time) observed with the CCD under these types of conditions. By what factor does the exposure time need to be increased in order to triple the S/N per pixel achieved with the CCD in this case?
[20 points]

Question 8: Cluster Structure & Dynamics

(a) Calculate the radial profile of the density, the circular speed, and the escape speed for a stellar system with a gravitational potential equal to

$$\Phi(r) = -4\pi G\rho_o a^2/[2(1+r/a)].$$

[35 points]

(b) A star cluster has a total energy equal to $-1.5\ M_\odot(\mathrm{km/s})^2$. Assuming that the total mass of the cluster is equal to $10^5\ M_\odot$ and that the mean stellar mass is equal to $0.8\ M_\odot$, calculate the mean kinetic energy of a star. Show all your calculations and equations used to obtain your answer.
[25 points]

(c) Discuss the effects of disk shocks on the stars in a star cluster as a function of the star distance from the cluster center. Discuss the possible effects of mass loss due to disk shocks (and the role of other dynamical processes in determining the disk shock effect) on the evolution the stellar mass function of a star cluster.
[40 points]

Question 9: Jupiters

(a) Consider a planet of mass M_p in circular orbit about a star of mass M_s at a distance d. Find an expression for the distance d_s of the center of the star from the center of mass of the star-planet system. Assume that $M_p \ll M_s$. Evaluate d_s for the Sun-Jupiter system ($M_J = 1.9 \times 10^{30}$ g, $M_\odot = 2.0 \times 10^{33}$ g, $d = 5.2$ AU, 1 AU = 1.5×10^5 km) and compare your result with the radius of the Sun ($R_\odot = 7.0 \times 10^5$ km).
[30 points]

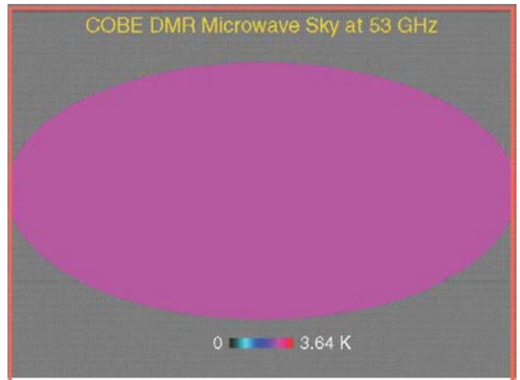
(b) Find an expression for the velocity v_s of the star's center of mass in the star-planet center-of-mass frame. Evaluate the value of v_s for the Sun-Jupiter system and discuss your result. Based on the expression for v_s, what sorts of planets are favored by searches for extrasolar planets that detect periodically varying Doppler shifts of the parent star? ($G = 6.7 \times 10^{-8} cm^3 g^{-1} s^{-2}$)
[40 points]

(c) Compare the spin angular momentum of the Sun with the orbital angular momentum of Jupiter. (You may make any simple assumption about the internal structure of the Sun for estimating its angular momentum.) Discuss the significance of your finding for explaining why the formation of planetary systems appears to be a common feature of star formation.
[30 points]

Question 10: Cosmology

(a) The Cosmological Principle and the Horizon problem

(i) The figure at right shows an all-sky map of the CMBR at a scale of 1 °K. How can you use this figure as evidence that the Cosmological Principle is valid? In your answer, be sure to state the Cosmological Principle.
[20 points]



1K scale

(ii) On what size scales is the Cosmological Principle valid? Cite evidence for your answer.
[10 points]

(iii) You can also use this figure to show that the Universe has a "horizon problem". What is the horizon problem? Use evidence from this figure to demonstrate that the Universe has a horizon problem.
[20 points]

(b) Friedmann's Equation

(i) Derive an expression for the variation of temperature as a function of time in the early universe when radiation dominates the energy density. Assume $\Lambda = 0$.
[15 points]

(ii) Derive an expression for the variation of temperature as a function of time when matter dominates the energy density for the case $k=0$. Assume $\Lambda = 0$.
[15 points]

(iii) Derive an expression for the variation of the temperature as a function of time if the cosmological constant dominates the energy density. Assume $\Lambda > 0$.
[15 points]

(c) In Big Bang nucleosynthesis, what is the "deuterium bottleneck"? What happens to "break" the deuterium bottleneck?
[5 points]