Ph.D. Qualifying Examination
Department of Astronomy
May 21, 2013
12:00 p.m. — 5:00 p.m.

Name: ___________________________

Student Number: _________________

The exam sheets are inside this envelope and are not fastened together. When you are finished, please put the questions and your answer sheets back in the envelope in the correct order. **Be sure the student number given to you by the proctor is on every page of your answers.**

Astronomy Program students **MUST** do the **FIRST TWO** problems and **SIX** more problems from the remaining **EIGHT**.

Astrophysics Program students **MUST** do **ONE** of the **FIRST TWO** problems and **FOUR** more problems from the remaining **EIGHT**. Astrophysics students must finish by **3:30 p.m.** (3.5 hours).

M.A. students must do **ONE** of the **FIRST TWO** problems and **THREE** more problems from the remaining **EIGHT**. M.A. students must finish by **2:30 p.m.** (2.5 hours).

Physical Constants

$c = 3.00 \times 10^{10}$ cm s^{-1}

$G = 6.67 \times 10^{-8}$ dyn cm^2 g^{-2}

$h = 6.63 \times 10^{-27}$ erg s

$k_B = 1.38 \times 10^{-16}$ erg K^{-1}

$m_p = 1.67 \times 10^{-24}$ g

$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}

$e = 4.80 \times 10^{-10}$ esu

$m_e = 9.11 \times 10^{-28}$ g

$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}

$\sigma_T = 6.65 \times 10^{-25}$ cm^2

1 eV $= 1.602 \times 10^{-12}$ erg

$R_\odot = 6.96 \times 10^{10}$ cm

$M_\odot = 1.99 \times 10^{33}$ g

$L_\odot = 3.90 \times 10^{33}$ erg s^{-1}

1 AU $= 1.496 \times 10^{13}$ cm

1 year $= 3.16 \times 10^7$ s

1 parsec $= 3.086 \times 10^{18}$ cm

$M_{V,\odot} = 4.83$ mag

$B.C._\odot = -0.07$ mag

$(B\text{–}V)_\odot = 0.64$ mag

$T_{eff,\odot} = 5770$ K

$M_\oplus = 5.97 \times 10^{27}$ g

$R_\oplus = 6.378 \times 10^8$ cm

REQUIRED: QUESTION 1: Star Clusters

Color-magnitude diagrams for two star clusters are shown on the following two pages. **Please include these pages with figures (with your work shown) in your assembled answers.**

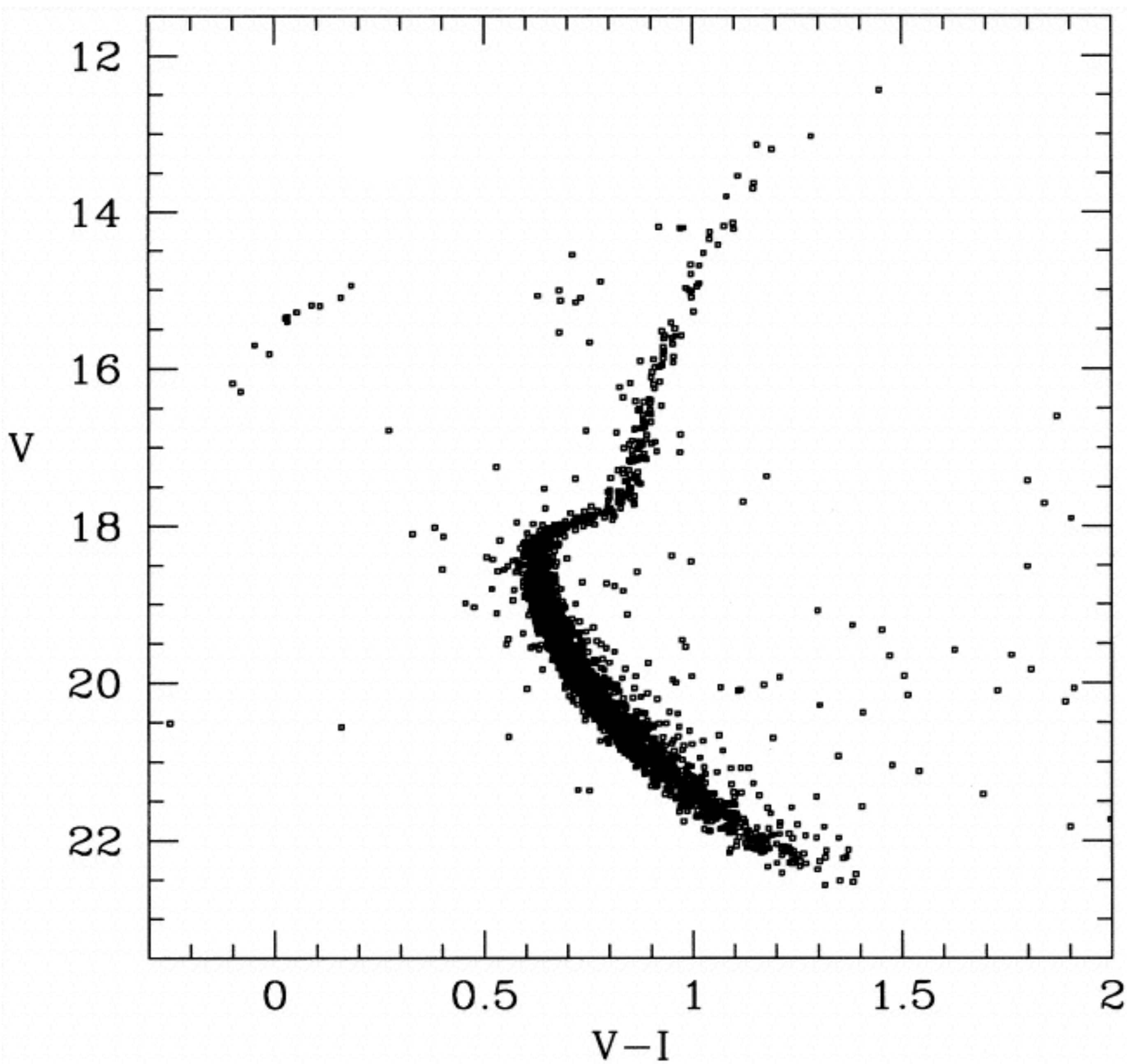
a) For each of these clusters, identify and label the major evolutionary stages seen in the diagrams (e.g. main-sequence, subgiant branch, red giant branch, etc, as appropriate). [25 points]

b) For each of these stages, describe the primary features that distinguish stars in these phases of evolution, including the dominant form of energy generation, relevant nuclear reactions, major structural features (e.g. location of convective or radiative regions, etc), and anything else you consider significant. [45 points]

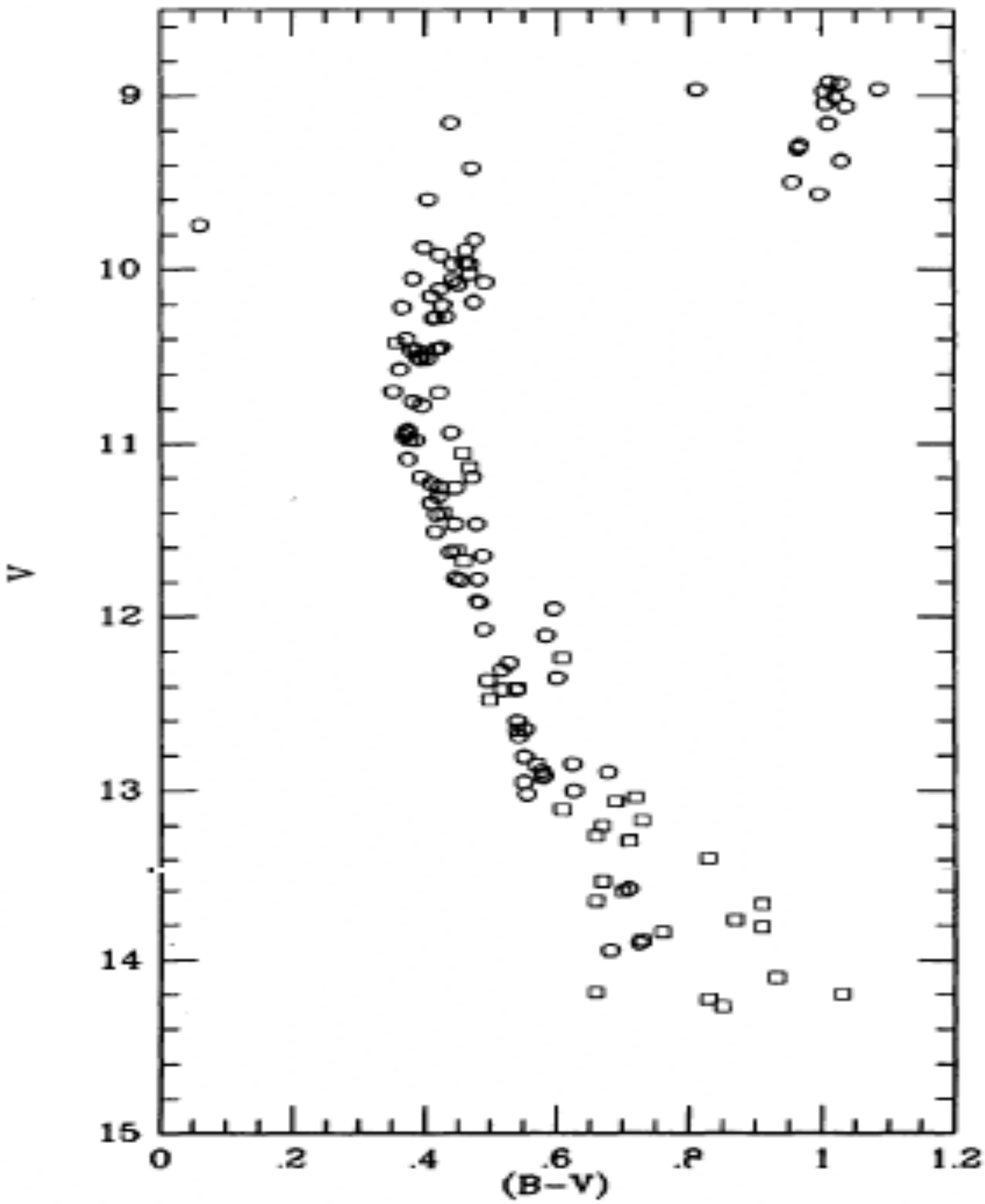
c) On the diagrams, indicate where the following kinds of stars would be, even if they are not present in the cluster and even if they fall outside the boundaries of the plot: RR Lyraes, Cepheids, white dwarf stars, blue stragglers. [10 points]

d) Give an estimate of the cluster age with an associated uncertainty, explaining clearly what features you used and your reasoning to derive the age. What additional information would you need to have to derive an accurate age for the cluster? [20 points]

Figures for Question 1 **Student Number ________**



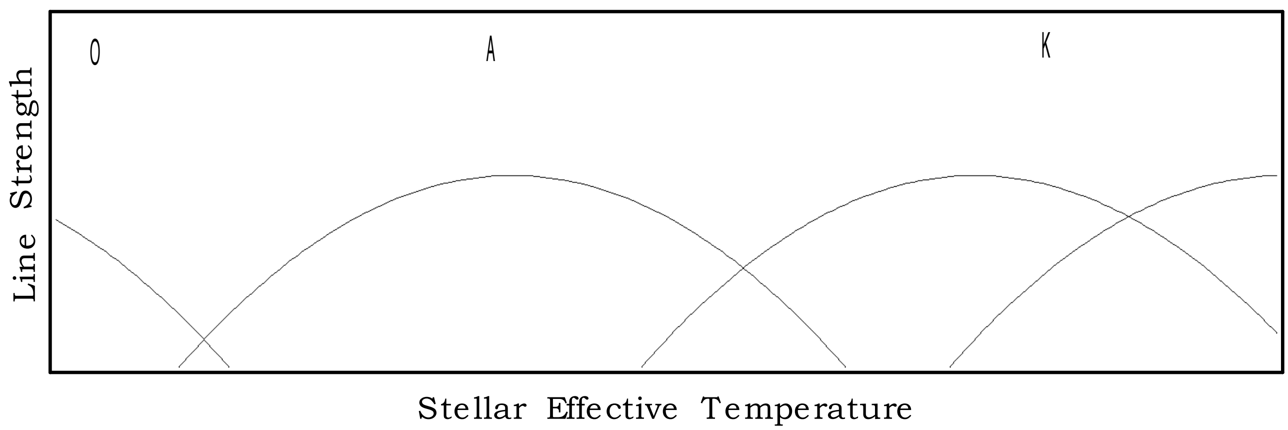
Figures for Question 1 **Student Number ________**



REQUIRED: QUESTION 2. General Astronomy

a) The diagram below shows (roughly) how the strength of certain types of absorption lines vary as a function of the surface temperature of a star. The four types of lines shown are for metals, hydrogen, helium, and molecules. A few of the major spectral types are also indicated. **Make a copy of this diagram on your answer sheet.**

i) Fill in the four missing spectral types. [8 points]
ii) Identify which curve represents which type of line. [12 points]



b) Beginning with the magnitude equation and with what you know about the definitions of apparent magnitude and absolute magnitude, derive the distance modulus equation (i.e., the equation defining *m*-*M*). Clearly define all quantities/symbols you use in your derivation. [20 points]

c) Describe forbidden lines and metastable levels, and explain why such lines occur in H II regions. The most important forbidden lines in H II regions come from which element? [20 points]

d) Explain why a main sequence dwarf evolves to become a giant. [40 points]

QUESTION 3. Spectroscopy

a) Draw a schematic of the setup of a typical spectrograph. Include basic components like the entrance aperture, collimating lens or mirror, dispersing element, camera, and detector. Briefly explain what purpose each of the components serves. Add any facts, definitions, or equations that you think are relevant. [40 points]

b) Consider a grating spectrograph and detector combination with the characteristics listed below. Suppose you are observing at second order and λ = 6200 Angstroms.

Grating: 600 lines/mm
Collimated beam width = 30 mm
Linear dispersion = 2.5 x 10^{-2} mm/Angstrom
2048x2048-pixel CCD with 15 micron pixels

i) Calculate both the resolving power, R, and the intrinsic spectral resolution of the spectrograph for this setup. [15 points]

ii) Calculate the minimum possible resolution set by the slit width (i.e., the resolution that is governed by the sampling theorem). [10 points]

iii) What is the effective spectral resolution of your spectrum, given the above spectrograph setup? Explain your answer. [10 points]

iv) Describe *one* science application (i.e., an observing project) for which the above spectrograph setup would be useful. Explain why the setup described here would work well for the specific technical requirements of the project. [25 points]

QUESTION 4. A Faint Companion to HD 118865

The UKIDSS Large Area Survey has recently identified a possible low mass companion to the 1.4 solar mass, F5V star HD 118865A. The separation between the two stars is 148". The companion is not an X-ray source, and is not detected in the UV, nor is it detected on the Palomar Sky Survey B or V plates, or in the 2MASS survey. What is known about the primary and secondary is summarized in the table below.

	HD 118865A	HD 118865B
Proper Motion (RA)	-96 mas yr^{-1}	
Proper Motion (Dec)	-49 mas yr^{-1}	
Parallax	16.02 mas	
[Fe/H]	+0.09	
Age	1.5-4.9 Gyr	
J Magnitude	6.98	18.08
H Magnitude	6.73	18.34
K Magnitude	6.67	18.34
Spectral Type	F5	
Log L_*/L_{Sun}	0.5	-5.24

a) Estimate the physical separation between HD 118865A and its companion in AU. [10 points]

b) Assuming the pair is gravitationally bound with an eccentricity of 0.5 (typical for binary stars), estimate lower and upper limits on the period of the orbit. State clearly any assumptions you make about the nature of the companion. [40 points]

c) Using the data provided in the table, estimate the probable temperature of the companion in each limiting case in part (b). [20 points]

d) What observations would allow you to determine the nature of the companion? Be specific. For example "take a spectrum" is not an acceptable answer. What spectral features would you look for and how would those spectral features distinguish the nature of the object. Likewise, "determine the SED" is not sufficiently specific. What new observations would you need and specifically, and what would you look for? Or do you have sufficient information already to rule out some of the options? If so, explain how. [30 points]

QUESTION 5. Stellar Populations

Consider the following observing programs designed to measure the properties of the stellar population in a dwarf galaxy of the Local Group. Assume the galaxy is at a distance of 500 kpc and suffers an insignificant amount of absorption along the line of sight to the galaxy.

a) Suppose your photometric observations in BVI reach to V~26 mag. What kinds of stars will you be able to observe? Give ranges or limits in terms of spectral type for both main-sequence and evolved stars. Approximately what mass ranges do these spectral types probe? You may find it helpful to sketch out the features of a color-magnitude diagram of the populations that might be seen. [30 points]

b) Based on photometric observations to this depth, what inferences will you be able to make about the possible star formation history in the dwarf galaxy? Be as specific as possible in terms of ages sampled and the limitations to possible star formation scenarios you can propose from these observations. [30 points]

c) Suppose you can also observe the brightest stars in the system spectroscopically and can derive selected elemental abundances. What elemental abundances would be most helpful to have to test your scenarios for star formation based on photometry? Explain why you choose these elements and what constraints these abundances would place on the star formation environment and history of enrichment in the galaxy. Explain your reasoning fully as you explore alternative scenarios. Feel free to use sketches of diagnostic diagrams for abundance ratios that might be helpful in detailing possible scenarios. [40 points]

QUESTION 6. Extragalactic Distances

a) What are “standard candles” and how are they used as distance indicators in astronomy? [25 points]

b) Give examples of **three** standard candles that are commonly used as extragalactic distance indicators in astronomy. In each case identify and explain the physical process that leads us to believe that these objects are indeed standard candles. [25 points]

c) What is the Tully-Fisher relation? How and why can the Tully-Fisher relation be used as a distance indicator? [25 points]

d) Define what is meant by the term "peculiar velocity" as it applies to galaxies. How are peculiar velocities measured? Describe how peculiar velocities can be derived, and explain how they can be used to derive an important cosmological parameter (Ω_m). [25 points]

QUESTION 7. Interacting Galaxies

For the following questions, consider an interacting system of galaxies identified with heliocentric velocities of 12075 km s^{-1} and 11925 km s^{-1}. Galaxy 1 has an apparent B-magnitude of 19.12, V-magnitude of 18.62, and an angular diameter of 3.51" at 25 mag/□. Galaxy 2 has an apparent B-magnitude of 19.65, V-magnitude of 19.25 and an angular diameter of 2.58" at 25 mag/□. The two galaxies are separated by 3.1". This region of the sky has an estimated A_B of 0.115 mag and A_V of 0.087 mag (Schlafly & Finkbeiner 2011).

a) What is the distance (in Mpc) to this system? [5 points]

b) If you observed this system on a night with really bad seeing (>3"), what would be the B and V magnitudes of the combined object you detect? [10 points]

c) Calculate the B-V colors for each galaxy and for the combined system. [10 points]

d) For each galaxy, calculate R_{25} in physical units (kpc). [10 points]

e) If you observed a similar system at a distance of 10 Mpc, what would be the angular extent of each galaxy? [10 points]

f) Calculate the luminosity of each galaxy (in $L_{\odot}$). [10 points]

g) Suppose you wish to complete a volume limited survey that includes these types of galaxies within a 10 Mpc volume. What is the limiting magnitude you will need for your observations? [5 points]

h) Estimate the total mass of the system. Clearly state your assumptions and show your work. [20 points]

i) Compare and contrast the mass-to-light ratio of this system with other measurements of mass-to-light ratios for extragalactic systems. Discuss the implications of your derived mass-to-light ratio. [10 points]

j) Given the above information, what is the likely morphological classification and approximate star formation history of these galaxies? Clearly justify your answer. [10 points]

QUESTION 8. Galactic Structure

a) Most spiral galaxies exhibit flat rotation curves (i.e., the rotation velocity of the gas and stars in the disk of the galaxy reaches some maximum value a few kiloparsecs from the galaxy center and stays at that value out to very large radius). Derive from simple dimensional arguments and Newton's laws
   i) the form of M(r) (the galaxy mass as a function of radius), and
   ii) ρ(r) (the density as a function of radius) implied by a flat rotation curve.
   [30 points]

b) How do your expressions for part (a) compare to what we know about the observed mass and/or density distribution of luminous matter in a typical spiral galaxy (e.g., like the Milky Way)? [10 points]

c) What do your answers to part (a) and (b) imply about the galaxies? [10 points]

d) The equation for the de Vaucouleurs ($r^{1/4}$) profile is:

$$I(r) = I_e 10^N, \text{ with } N = -3.331[(r/r_e)^{1/4} - 1]$$

where I_e is the surface brightness at the effective radius r_e.

   i) Derive the formula for the surface brightness in magnitude units ($\mu(r)$ in mag/□) for the de Vaucouleurs profile. Your final expression should be written in terms of μ_e and r_e. [15 points]

   ii) Calculate the central surface brightness of a galaxy that follows a de Vaucouleurs law and has μ_e = 21.0 mag/□. [15 points]

   iii) Assume that an E0 galaxy has a total B-band apparent magnitude B = 11.85 and an effective radius r_e = 23 arcsec. What is the mean surface brightness (in mag/□) inside the effective radius? [20 points]

QUESTION 9. Galactic Dynamics

a) Calculate the density profile and the total mass of a spherical stellar system that generates a potential $\Phi(r) = -\frac{GM}{r_J}\ln\left(\frac{r+r_J}{r}\right)$. Calculate the scaling of the circular velocity with r for $r << r_J$ and for $r >> r_J$. [25 points]

b) Discuss how the global and local (i.e. measured at different distances from the cluster center) stellar mass function of a star cluster may evolve during the cluster long-term evolution driven by the effects of two-body relaxation. In your discussion, identify the key processes responsible for the mass function evolution. [25 points]

c) Write the collisionless Boltzmann equation and the Fokker-Planck equation: discuss their physical meaning and differences, the relevant timescales and examples of stellar systems whose evolution can be described by each of these equations. [30 points]

d) In a stellar system, stars are spatially distributed according to a King model and their total potential energy is equal to (in some arbitrary units) –0.5. [20 points]

(i) Assuming velocities are distributed according to the appropriate King distribution function and the system is in dynamical equilibrium, what is the total kinetic energy of the system (in the same arbitrary units)? Explain your answer.

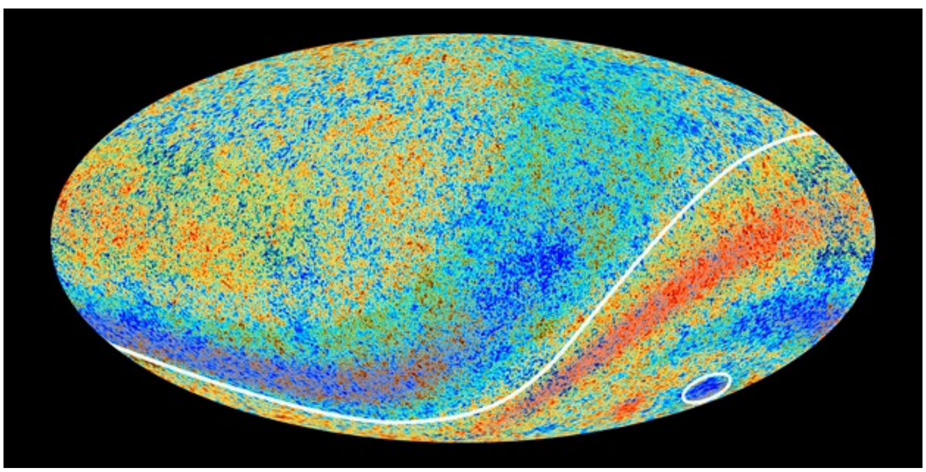
(ii) Describe the response of the system if all stars instantaneously lose a fraction f of their mass.

QUESTION 10. Results from the Planck Mission

a) The Planck Mission is reporting that the Hubble constant is $H_0 = 67.4 \pm 1.4$ km s^{-1}/Mpc and the age of the Universe is 13.8 By. Typically, how do astronomers estimate the age of the Universe from the Hubble constant? Use a diagram that plots *R(t)* (the scale factor in the FRW metric) vs *t* for cosmological models without Dark Energy to explain why this calculation has traditionally overestimated the age. Previously, WMAP estimated the age to be 13.7 By. How does Planck's lower estimate for the fraction of Dark Energy in the Universe (but leaving *k=1*) lead to an older Universe? [40 points]

b) The CMBR photons mapped by Planck do not originate at the Big Bang, but rather "break out" 380,000 years afterwards. Why can't we see photons originating in the Big Bang? What is the physical condition that must be met for this "break out" to occur? What is the temperature of the Universe when the break out occurs? Why is this temperature different from the naïve expectation? (What is expected?) [30 points]

c) The Planck mission team also reports that there is an asymmetry in the average temperatures on opposite hemispheres of the sky, with slightly higher average temperatures in the southern ecliptic hemisphere, and a cosmic cold spot (circled). (red – hotter; blue – colder)



Cosmology is formulated with the Cosmological Principle explicitly built in. What is the Cosmological Principle? How does this discovery apparently violate the Cosmological Principle? How is the Cosmological Principle built into the FRW metric,

$$ds^2 = dt^2 - R^2(t)[d\chi^2 + S_k^2(\chi)(d\theta^2 + \sin^2\theta\, d\phi^2)]$$

used in the formulation of the Friedmann equations? Speculate: How might this discovery be explained so that the Cosmological Principle remains valid? [30 points]