# Ph.D. Qualifying Examination
# Department of Astronomy
# June 5, 2012
# 8:30 a.m. — 1:30 p.m.

**Name: ___________________________**

**Student Number: _________________**

The exam sheets are inside this envelope and are not fastened together. When you are finished, please put the questions and your answer sheets back in the envelope in the correct order. **Be sure the student number given to you by the proctor is on every page of your answers.**

Astronomy Program students **MUST** do the **FIRST TWO** problems and **SIX** more problems from the remaining **EIGHT**.

Astrophysics Program students **MUST** do **ONE** of the **FIRST TWO** problems and **FOUR** more problems from the remaining **EIGHT**. Astrophysics students must finish by **12:00 p.m.** (3.5 hours).

M.A. students must do **ONE** of the **FIRST TWO** problems and **THREE** more problems from the remaining **EIGHT**. M.A. students must finish by **11:00 a.m.** (2.5 hours).

## Physical Constants

$c = 3.00 \times 10^{10}$ cm s$^{-1}$

$G = 6.67 \times 10^{-8}$ dyn cm$^2$ g$^{-2}$

$h = 6.63 \times 10^{-27}$ erg s

$k_B = 1.38 \times 10^{-16}$ erg K$^{-1}$

$m_p = 1.67 \times 10^{-24}$ g

$a = 7.56 \times 10^{-15}$ erg cm$^{-3}$ K$^{-4}$

$e = 4.80 \times 10^{-10}$ esu

$m_e = 9.11 \times 10^{-28}$ g

$\sigma = 5.67 \times 10^{-5}$ erg cm$^{-2}$ K$^{-4}$ s$^{-1}$

$\sigma_T = 6.65 \times 10^{-25}$ cm$^2$

1 eV $= 1.602 \times 10^{-12}$ erg

$R_\odot = 6.96 \times 10^{10}$ cm

$M_\odot = 1.99 \times 10^{33}$ g

$L_\odot = 3.90 \times 10^{33}$ erg s$^{-1}$

1 AU $= 1.496 \times 10^{13}$ cm

1 year $= 3.16 \times 10^7$ s

1 parsec $= 3.086 \times 10^{18}$ cm

$M_{V,\odot} = 4.83$ mag

$B.C._\odot = -0.07$ mag

$(B\text{–}V)_\odot = 0.64$ mag

$T_{eff,\odot} = 5770$ K

$M_\oplus = 5.97 \times 10^{27}$ g

$R_\oplus = 6.378 \times 10^8$ cm

**REQUIRED: QUESTION 1. Simple Stellar Populations**

a) Consider a cluster of stars that contains

    100 stars with absolute magnitude $M_V$=1.0,
    1000 stars with $M_V$=4.0, and
    10000 stars with $M_V$=7.0.

What is the absolute magnitude of the cluster taken as a whole? What would be the apparent magnitude of the star cluster if it were located at a distance of 3.0 kpc with foreground extinction, $A_V$, of 0.3? [20 points]

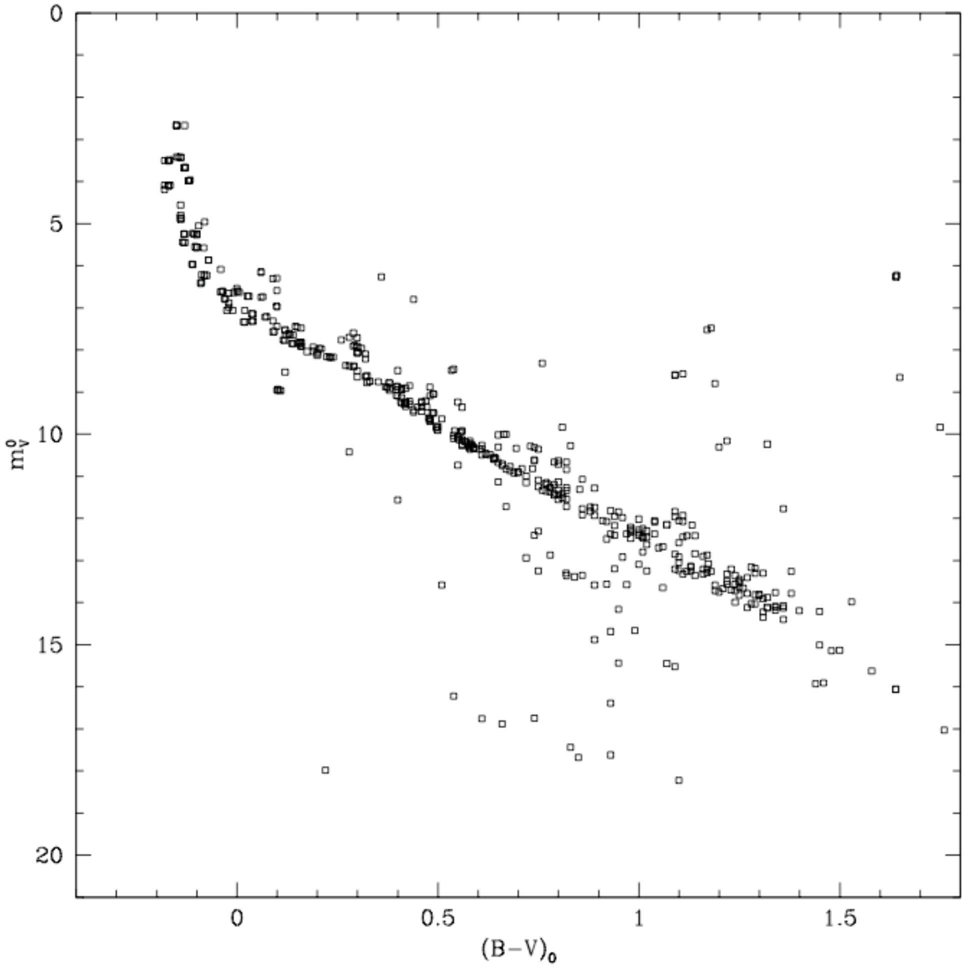
b) Using the data presented on the next page, (i) derive a distance modulus and (ii) calculate the corresponding distance (in parsec) to this star cluster. [20 points]

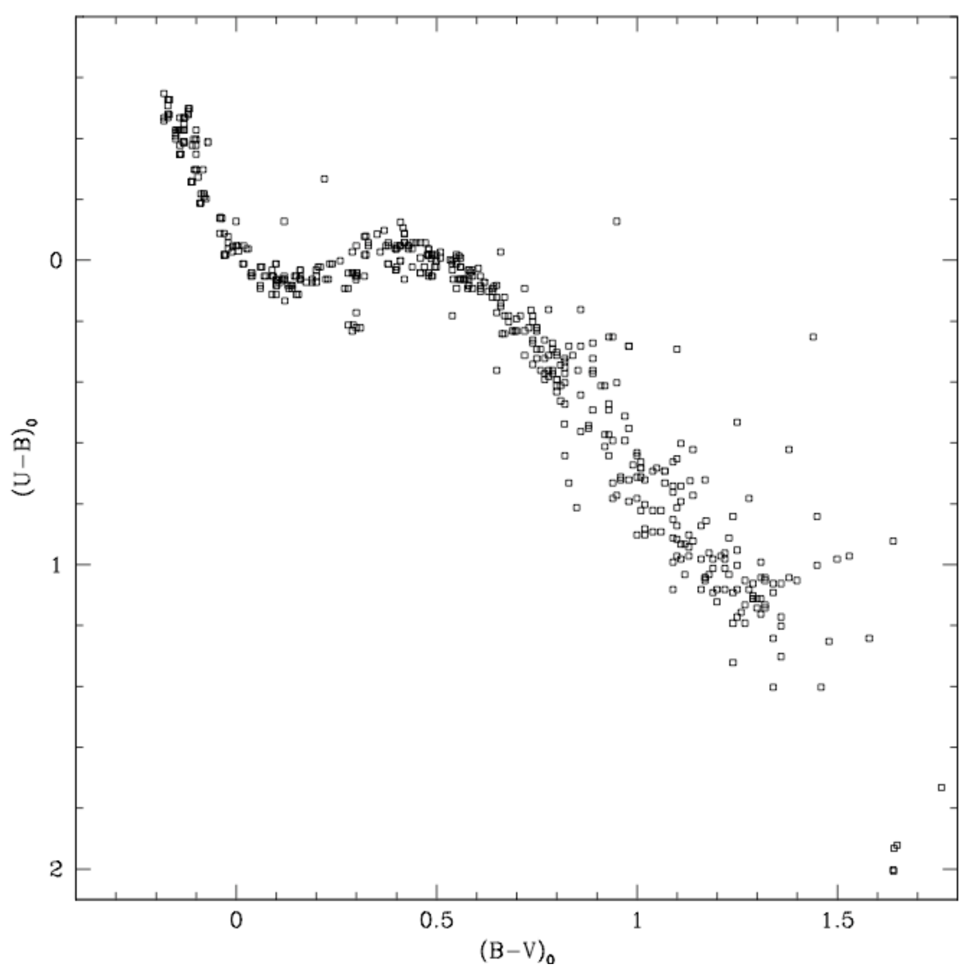
c) Estimate the age of the star cluster presented in (b). Clearly specify all assumptions. [20 points]

d) Calculate the expected linewidth of the MgIb line (517.6 nm) if the star cluster is located in a potential well of $10^6$ $M_\odot$ within 10 pc. Similarly, calculate the expected linewidth of MgIb for a star cluster located in a potential well of $10^{12}$ $M_\odot$ within 50 kpc. Clearly specify all assumptions. [20 points]

e) Consider an interstellar cloud with a temperature of 80 K and neutral hydrogen density of $n_H = 10^8$ $m^{-3}$ at a distance of 100 pc. If the cloud has a radius of $10^{16}$ m, (i) how many neutral hydrogen atoms are in the cloud? (ii) What is the mass of the cloud (in units of $M_\odot$)? (iii) What is the approximate free-fall time of the cloud? (iv) What is the gravitational potential energy of the cloud? [20 points]

**Figures for Question 1(b) and 1(c)**



**REQUIRED: QUESTION 2. Astronomy for the Public**

Astronomy captures the public's imagination, and an important part of an astronomer's job is to discuss astronomy with non-expert or semi-expert members of the public. You may have done such "outreach" at Kirkwood Observatory. Satisfying the public's thirst for astronomical knowledge can be fun and rewarding, but also challenging when you're addressing misconceptions.

Pretend you're in a public forum, for example, a Kirkwood Open House, and you're asked the questions listed below. **Answer four of these [25 points each].** You will be graded on the correctness, thoroughness, and effectiveness of your response.

a) I read that liquid water is required for life to develop, is this true? What other places in the solar system (besides Earth) have liquid water? Could they harbor life? Wasn't water discovered during the last few years in a very surprising place?

b) I read that when the Apollo 11 astronauts left Earth orbit, they did not need to fire their spacecraft's engines for more than just a fraction of an hour, and yet the astronauts subsequently coasted all the way to the Moon. How is that possible?

c) How can the Universe be expanding? What is it expanding into?

d) Astronomers say that the Universe is not just 5000 years old, but rather that it is *billions* of years old! How do you know this? Convince me!

e) What are black holes? Are there different kinds? Where do they come from? Are we in any danger from a black hole?

f) I heard that, even though we can't see inside the Sun, we know about its interior in great detail! Describe the interior of the Sun to me. Where did the Sun come from? How long will it exist?

g) When I go to the dark countryside, I can see that great white band going across the sky, called the Milky Way. I read that the Milky Way is made up of an unbelievable number of stars. Tell me what the Milky Way is. Are there other Milky Ways?

h) Today (June 5, 2012), an important, recurring astronomical event is taking place, that anyone with an amateur telescope can observe. This event will take place again at a precisely calculated time in the next century. Identify the event, describe it, and discuss its historical importance for astronomy.

**QUESTION 3. The Early Universe**

a) Write out Friedmann's Equation in the form appropriate for the early universe. Be sure to explain/motivate the assumptions you make in deriving the equation. [20 points]

b) Suppose a new charged lepton was discovered at CERN – the cruton – with mass $m_{cr}$ = 120 MeV. The anti-cruton was seen soon thereafter. No corresponding cruton neutrino has yet been detected. What is the value of $g_\star$ when $\gamma$'s, $\nu$'s, e's, $\mu$'s, and crutons are driving the expansion? [20 points]

c) Solve Friedmann's Equation for R = R(t). Translate your solution to T = T(t). [20 points]

d) As the Universe expands and T falls below $k_B T = m_{cr} c^2$, the crutons become nonrelativistic and annihilate. Find $T_1/T_2$, the ratio of the TE temperature of the relativistic particles before annihilation, $T_1$, to the temperature after annihilation, $T_2$. [20 points]

e) Measurements show that today $\Omega^0{}_m = 0.24$ and $\Omega^0{}_\gamma = 4.75 \times 10^{-5}$, or today the universe is "matter dominated." But the early universe was "radiation dominated." At what redshift did this transition occur? What was the temperature of the CMBR when that transition occurred? [20 points]

**QUESTION 4. Spectroscopy**

Answer each the following questions clearly and thoroughly, using words, equations, and/or illustrations.

a) What is a radial velocity? How is it defined, mathematically? How does one measure a radial velocity for a star? [20 points]

b) Explain how one can measure a projected rotation velocity, v sin(*i*), for a star. Explain what *i* is in the previous equation and why the measured quantity is the product v sin *i*. [10 points]

c) Explain how one can use observations of star-forming regions in a spiral galaxy to derive the galaxy's rotation curve. Be specific about what is being observed, and how the observations are obtained. Sketch a typical rotation curve for a massive spiral galaxy derived in this way. Label the axes with reasonable values. [40 points]

d) Finally, explain clearly and specifically how single-dish radio observations of galaxies can be used to derive the following four quantities: radial velocity; rotation velocity; neutral gas content; and dynamical mass. [30 points]

**QUESTION 5. Oxygen**

Consider the case of an ionized gas with a specific electron temperature ($T_e$) and density ($n_e$). Answer the following, assuming that all upward transitions in the relevant atoms are due to collisional excitation.

a) Draw the energy level diagram for singly ionized oxygen ($O^+$). Include the three lowest-lying energy states only, and include the term and level designations. Label the downward transitions from the upper state(s) to the lower state(s) with the wavelengths of the astrophysically relevant transitions. [20 points]

b) Explain how the electron density ($n_e$) can be inferred from the emission-line flux ratio of specific $O^+$ lines. That is, give a detailed explanation of the physics behind the measurement of $n_e$ using a pair of $O^+$ emission lines. Make it clear why this particular pair of emission lines is density sensitive. Give characteristic values for the ratio and the corresponding densities in the low- and high-density limits. [30 points]

c) Draw the energy level diagram for doubly ionized oxygen ($O^{++}$). Include the five lowest-lying energy states only and include the term and level designations. Label the downward transitions from the upper state(s) to the lower state(s) with the wavelengths of the astrophysically relevant transitions. [20 points]

d) Explain how the electron temperature ($T_e$) can be inferred from the emission-line flux ratio of specific $O^{++}$ lines. That is, give a detailed explanation of the physics behind the measurement of $T_e$ using $O^{++}$ emission lines. Make it clear why this particular set of emission lines is temperature sensitive. Give characteristic values for the ratio (approximate values are fine) and the corresponding temperatures that are observed in real nebulae. [30 points]

**QUESTION 6. Stellar Evolution**

a) Describe how the properties of stars vary along the main sequence from the lowest to the highest mass stars. You may find it helpful to draw out an HR diagram with properties labeled in different mass ranges. Your answer should include all of the following items: [50 points]

   i) equation of state and pressure support, nuclear reactions, opacity sources

   ii) parameters for the internal structure: convective and radiative regions, typical central densities and temperatures

   iii) variations in mass (M), radius (R), luminosity (L), and effective temperature ($T_{eff}$) along the main sequence

b) On a single diagram, sketch the evolutionary tracks for a **1 solar mass star** and a **6 solar mass star** (assuming solar metallicity) from the ZAMS to its final remnant state. Label the axes with appropriate numerical values. Along each track, indicate the major evolutionary stages and describe what distinguishes them, including such aspects as dominant nuclear reactions, energy generation, major structural changes, and anything else you consider significant that would distinguish the behavior at the two different masses. [50 points]

**QUESTION 7. Stellar Atmospheres**

a) List four basic assumptions made in the description and modeling of stellar atmospheres. Define each clearly but succinctly. For each of these assumptions give
   i) an example of a type of star for which the basic assumption is valid and explain why and,
   ii) an alternative example of a type of star or situation in which the assumption is not valid and explain why not. [25 points]

b) Spectra of two stars are given on the following page. For each of these
   i) Identify and label as many spectral and continuum features in the observed flux distribution as you can. [15 points]

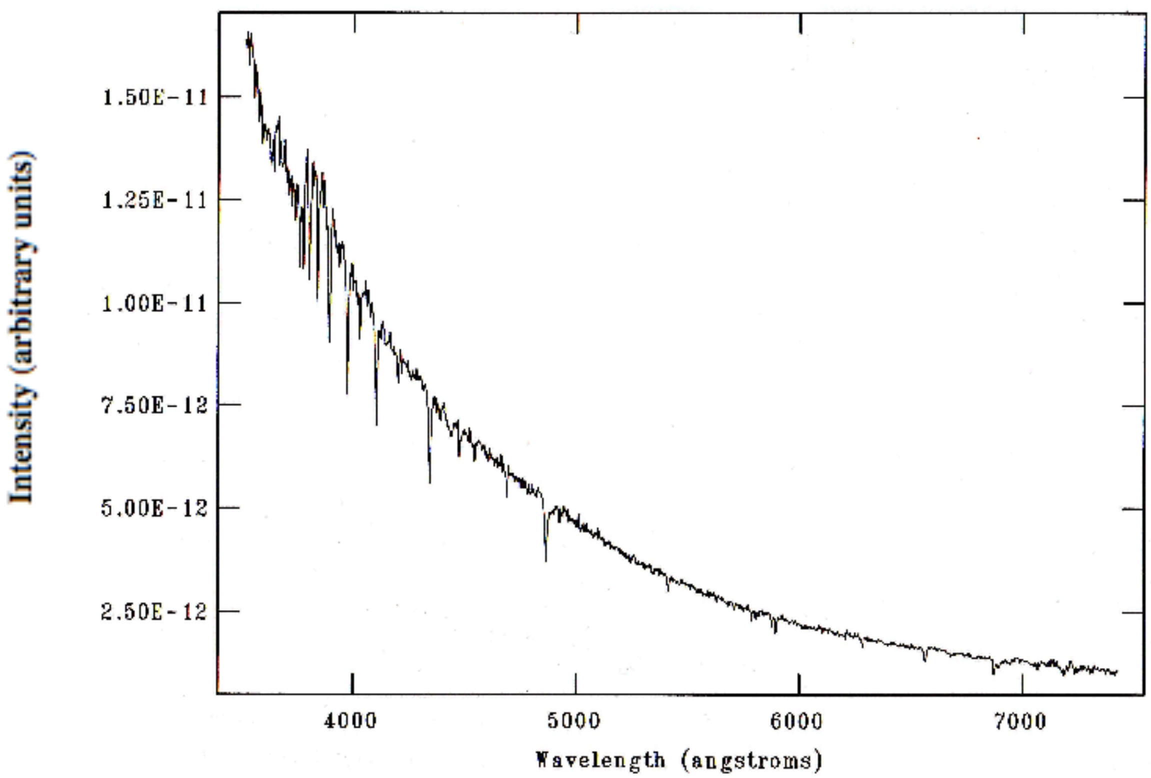
   ii) Derive an estimate of the spectral type (sub-types are not necessary) and the effective temperature for each star, and describe what information you used to obtain your result. [10 points]

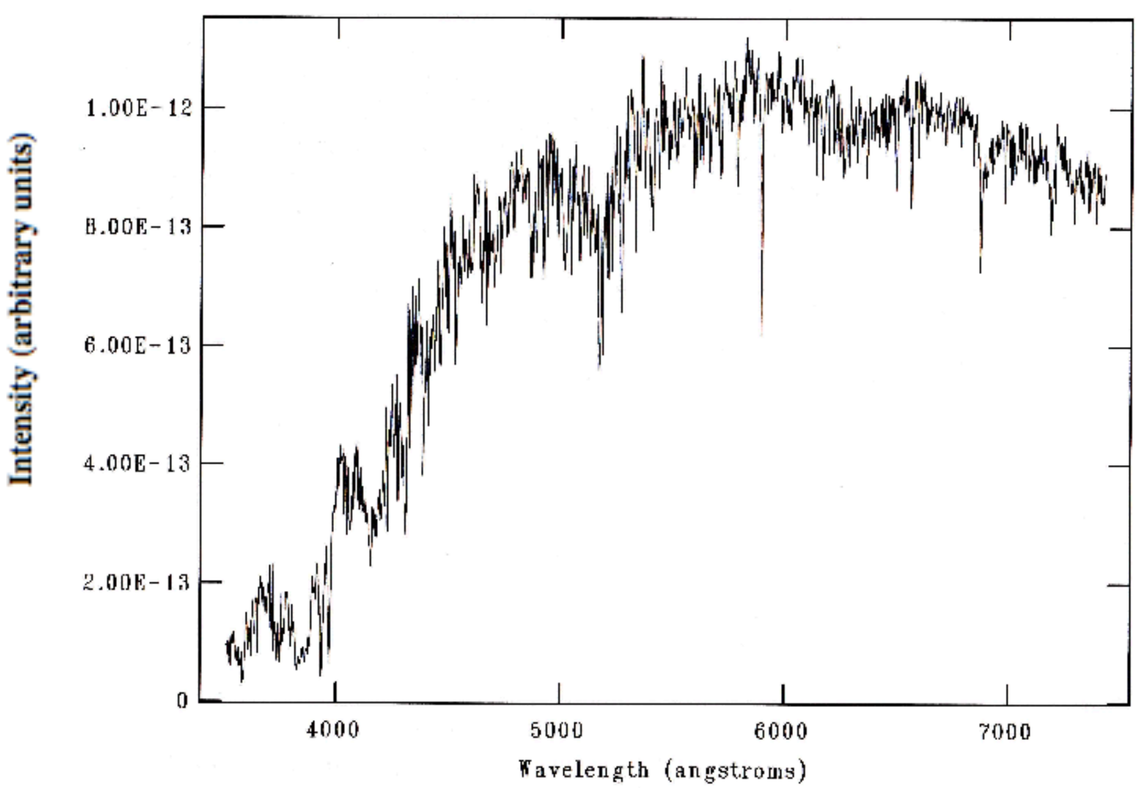
   iii) Identify the most important source of opacity in the optical region of the spectrum and describe how the dominant opacity source would vary in other spectral regions not shown (e.g. the UV and IR). [10 points]

c) Explain how the Balmer lines vary with stellar spectral type. At what effective temperature do they reach their peak strength, and describe what physical effects control where their peak occurs. [20 points]

d) What factors determine how strong a spectral line will be? Which will generally have the strongest effect on line strength and why? Describe at least three mechanisms that lead to the broadening of spectral lines. [20 points]

**Figures for Question 7(b)**



**QUESTION 8. Galaxies**

Consider the four main morphological types of galaxies in Hubble's original classification system: elliptical, spiral, lenticular (S0), and irregular.

Compare and contrast all four of these galaxy types in terms of the properties listed below. Feel free to use tables, plots, and/or sketches to supplement your written answer. Also use numerical values or quantitative examples wherever possible. **[10 points for each property.]**

a) morphology

b) surface brightness profile

c) (B-V) color

d) star formation rate

e) central surface brightness

f) stellar kinematics

g) stellar populations and mean stellar ages

h) neutral gas content

i) molecular gas content

j) metallicity

**QUESTION 9. Dynamics**

Observations made over the past 80 years have lead us to believe that most research efforts in astronomy are devoted to only a small portion of the mass content of the universe and that most of the mass is unseen. This belief stems from discrepancies between observed mass and inferred mass. The following questions deal with the observational and theoretical foundations of "missing mass" or "dark matter".

a) Two studies carried out in the early 1930s, one by Jan Oort involving the Milky Way and the other by Fritz Zwicky involving the Coma Cluster, were the first to reveal this mass discrepancy. Choose one of these studies and explain the observations and their implications. Provide a <u>simple</u> mathematic description of the problem. [20 points]

b) This problem spread to spiral galaxies in the 1960s, when both observational and theoretical evidence surfaced supporting the idea that most of the mass in spiral galaxies is not visible. Describe the observational evidence and explain why they presented a problem. Provide a <u>simple</u> mathematic description of the problem. [20 points]

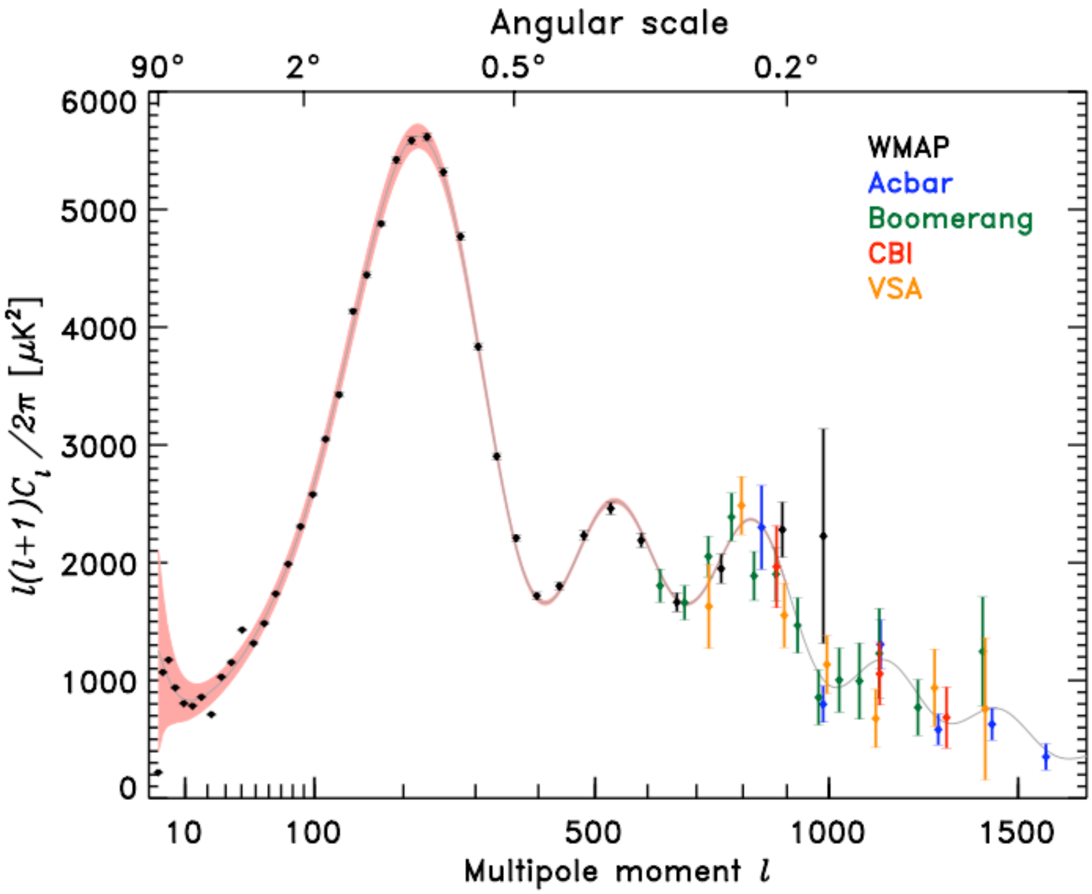
c) Building on part b, calculate the mass interior to 30 kpc in a typical spiral galaxy using numbers appropriate for such galaxies. [20 points]

d) By the early 1980s, observations permitted by improved satellite-based observational capabilities suggested that the dark matter problem was not limited to spirals but rather many elliptical galaxies also possess large amounts of dark matter. Describe these observations and their implications for dark matter. [20 points]

e) While the identity of dark matter is not known, two broad classes of candidates have been proposed. Name and briefly describe these two classes. [20 points]

**QUESTION 10. Structure of the Universe**

The following figure is fundamental to our understanding of the structure of the universe.




a) Explain what this figure is showing. Your answer should include a clear explanation of the units of both x-axes and the y-axis. [25 points]

b) Describe the observations required to create this figure. Your answer should also include an explanation for why multiple experiments are required to create this figure. [25 points]

c) Briefly comment on the technical innovations within the last decade that have impacted our ability to make these measurements. [10 points]

d) Provide a scientific interpretation of the various features in this figure. Your answer should include both a description of the physical signature and the corresponding conclusion about structure in the universe. [40 points]

$$ds^2 = dt^2 - R^2(t)[d\chi^2 + S_k^2(\chi)(d\theta^2 + \sin^2\theta\, d\phi^2)]$$

$$\chi(z) = H_0 \int_0^z dz'/H(z')$$

$$\frac{\dot{R}^2}{R^2} - \frac{8\pi G\rho}{3} = -\frac{kc^2}{R^2} + \Lambda/3$$

$$H = (\frac{\dot{R}}{R})$$

$$H^2(R) = H_0^2[\Omega_m^0(R_0/R)^3 + \Omega_r^0(R_0/R)^4 + \Omega_\Lambda^0 + (1-\Omega_T^0)(R_0/R)^2]$$

$$\frac{R_0}{R} = 1 + z$$

$$t(z) = \frac{1}{H_0}\int_0^{1/(1+z)} \frac{dx}{x\sqrt{\Omega_\Lambda + \Omega_m^0 x^{-3} + \Omega_r^0 x^{-4} + (1-\Omega_T^0)x^{-2}}}; \quad x = 1/(1+z)$$

$$\Omega = \rho/\rho_c$$

$$\rho_c = 3H^2/8\pi G$$

$$\rho_c^0 = \frac{3H_0^2}{8\pi G} = 1.88 \times 10^{-29} h^2 \text{ gm/cm}^3$$

$$D_L = (1+z)R_0 S_k(\chi) \quad \text{(luminosity)}$$

$$D_A = (1+z)^{-1} R_0 S_k(\chi) \quad \text{(angular diameter)}$$

$$\epsilon_\gamma = \rho_\gamma c^2 = a_B T^4 = 3p_\gamma$$

$$n_{rel} = \frac{1.2}{\pi^2} g_{\star n}(k_B T/\hbar c)^3$$

$$g_{\star n} = \sum_{\text{bosons}} g_i + \frac{3}{4}\sum_{\text{fermions}} g_i$$

$$\epsilon_{rel} = \rho_{rel}c^2 = \frac{\pi^2}{30} g_\star (k_B T)(k_B T/\hbar c)^3$$

$$s = \frac{2\pi^2}{45} g_\star (k_B T/\hbar c)^3$$

$$g_\star = \sum_{\text{bosons}} g_i + \frac{7}{8}\sum_{\text{fermions}} g_i$$

$$H_0 = 72 \text{ km/s/Mpc}$$