Ph.D. Qualifying Examination
Department of Astronomy
June 8, 2011
8:30 a.m. — 1:30 p.m.

Name: ________________________

Student Number: ________________

The exam sheets are inside this envelope and are not fastened together. When you are finished, please put the questions and your answer sheets back in the envelope in the correct order. **Be sure the student number given to you by the proctor is on every page of your answers.**

Astronomy Program students **MUST** do the **FIRST TWO** problems and **SIX** more problems from the remaining **EIGHT**.

Astrophysics Program students **MUST** do **ONE** of the **FIRST TWO** problems and **FOUR** more problems from the remaining **EIGHT**. Astrophysics students must finish by **12:00 p.m.** (3.5 hours).

M.A. students must do **ONE** of the **FIRST TWO** problems and **THREE** more problems from the remaining **EIGHT**. M.A. students must finish by **11:00 a.m.** (2.5 hours).

Physical Constants

$c = 3.00 \times 10^{10}$ cm/s
$G = 6.67 \times 10^{-8}$ dyn cm^2/g^2
$h = 6.63 \times 10^{-27}$ erg s
$k = 1.38 \times 10^{-16}$ erg/K
$m_p = 1.67 \times 10^{-24}$ g
$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}
$e = 4.80 \times 10^{-10}$ esu
$m_e = 9.11 \times 10^{-28}$ g
$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
$\sigma_T = 6.65 \times 10^{-25}$ cm^2
1 eV = 1.6×10^{-12} erg

$R_\odot = 6.96 \times 10^{10}$ cm
$M_\odot = 1.99 \times 10^{33}$ g
$L_\odot = 3.90 \times 10^{33}$ erg/s
A.U. = 1.50×10^{13} cm
1 year = 3.16×10^7 s
1 parsec = 3.09×10^{18} cm
$M_{V\odot} = 4.83$ mag
$B.C._\odot = -0.07$ mag
$(B-V)_\odot = 0.64$ mag
$T_{eff,\odot} = 5770$ K
$M_E = 5.97 \times 10^{27}$ g
$R_E = 6.38 \times 10^8$ cm

$$ds^2 = dt^2 - R^2(t)[d\chi^2 + S_k^2(\chi)(d\theta^2 + \sin^2\theta \, d\phi^2)]$$

$$S_k(\chi) = \begin{cases} \sin\chi & \text{k} = +1 \\ \chi & \text{k} = \ \ 0 \\ \sinh\chi & \text{k} = -1 \end{cases}$$

$$\chi(z) = H_0 \int_0^z dz'/H(z')$$

$$(\frac{\dot{R}}{R})^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R^2} + \Lambda/3$$

$$H = (\frac{\dot{R}}{R})$$

$$H^2(R) = H_0^2[\Omega_\Lambda^0 + \Omega_m^0(R_0/R)^3 + \Omega_r^0(R_0/R)^4 + (1 - \Omega_T^0)(R_0/R)^2]$$

$$\frac{R_0}{R} = 1 + z$$

$$H^2(z) = H_0^2[\Omega_\Lambda + \Omega_m^0(1+z)^3 + \Omega_r^0(1+z)^4 + (1 - \Omega_T^0)(1+z)^2]$$

$$t(z) = \frac{1}{H_0}\int_0^{1/(1+z)} \frac{dx}{x\sqrt{\Omega_\Lambda + \Omega_m^0 x^{-3} + \Omega_r^0 x^{-4} + (1 - \Omega_T^0)x^{-2}}}; \quad x = 1/(1+z)$$

$$\Omega = \rho/\rho_c$$

$$\rho_c = 3H^2/8\pi G$$

$$D = R_0\chi \quad \text{(comoving)}$$

$$D_P = R(t)\chi \quad \text{(proper)}$$

$$D_L = (1+z)R_0 S_k(\chi) \quad \text{(luminosity)}$$

$$D_A = (1+z)^{-1} R_0 S_k(\chi) \quad \text{(angular diameter)}$$

$$\rho_c^0 = \frac{3H_0^2}{8\pi G} = 1.88 \times 10^{-29} h^2 \ \text{gm/cm}^3$$

$$\epsilon_\gamma = a_B T^4 = 3p_\gamma$$

$$\epsilon_{rel} = \rho_{rel}c^2 = \frac{\pi^2}{30} g_\star (k_B T)(k_B T/\hbar c)^3$$

$$s = \frac{2\pi^2}{45} g_{\star s}(k_B T/\hbar c)^3$$

$$g_\star = g_{\star s} = \sum_{\text{bosons}} g_i + \frac{7}{8}\sum_{\text{fermions}} g_i$$

REQUIRED: QUESTION 1. Major Figures in Astronomy

a) Fill in the H-R Diagram provided on the following page. First, draw and label the main sequence and the other M-K Luminosity classes. Second, on the x-axis, indicate: (i) all of the major spectral classes and (ii) 3-4 well-spaced values of the corresponding effective temperature. Third, provide representative absolute magnitudes and luminosities on the relevant y-axis. Fourth, draw in lines of constant radius for $100R_{\odot}$, $1R_{\odot}$, and $0.01R_{\odot}$. [40 pts]

b) Sketch the expected rotation curves for (i) the solar system; (ii) a low mass dwarf irregular galaxy; (iii) a giant spiral galaxy. Be sure to include 3-4 well-spaced representative values on both axes. [21 pts]

c) Sketch the expected optical surface brightness profiles for (i) elliptical galaxies and (ii) spiral galaxies. Be sure to include 3-4 well-spaced representative values on both axes. [14 pts]

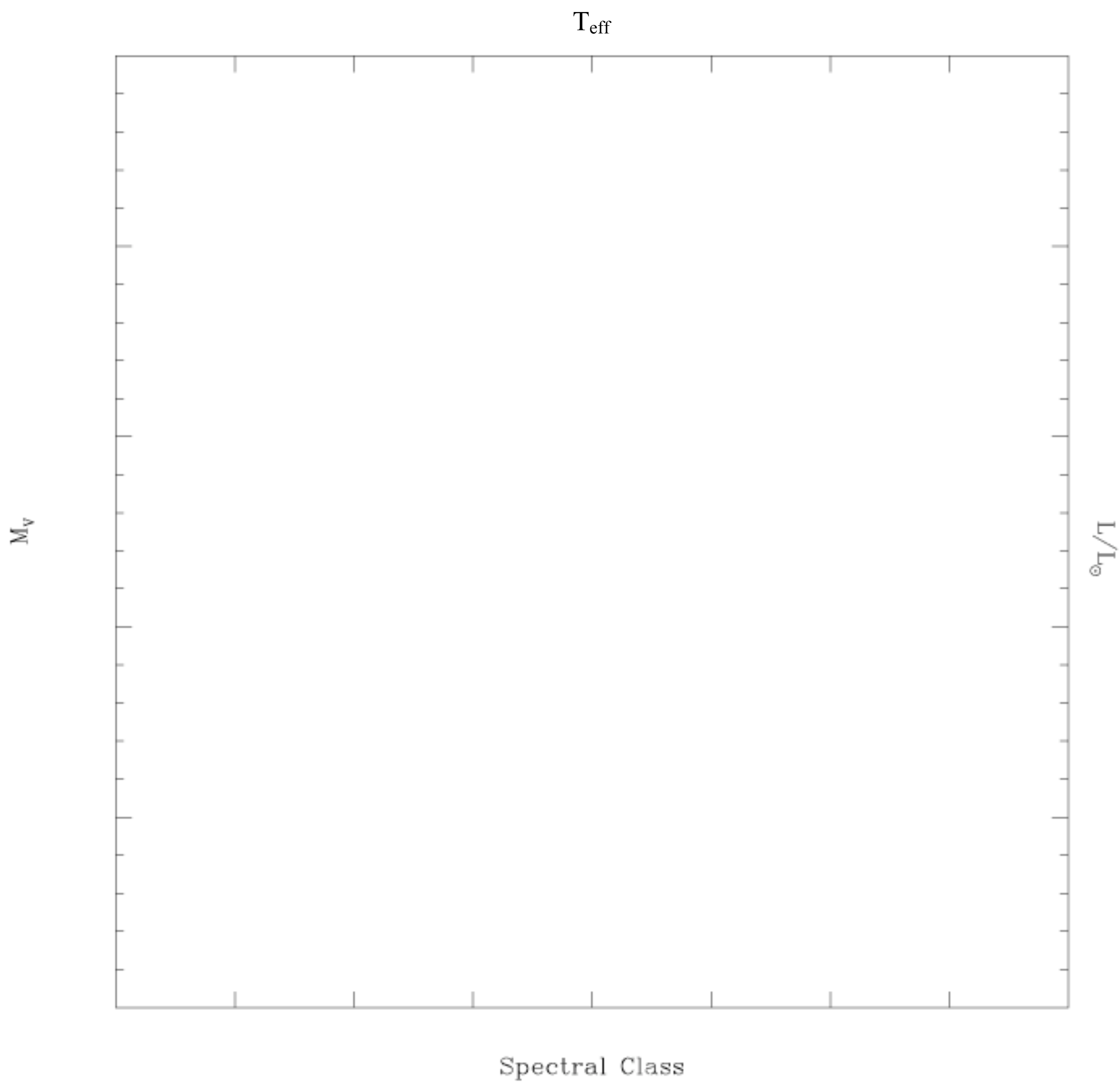
d) Sketch the generic form of the luminosity function for galaxies. Note major features in your sketch and clearly denote 3-4 well-spaced representative values on the x-axis. [7 pts]

e) Draw a diagram which illustrates the geometric configuration of the Earth-Moon-Sun for the 8-major moon phases. Clearly denote the orientation and label each phase. What are the approximate rise, transit, and set times for a waning gibbous moon? [18 pts]

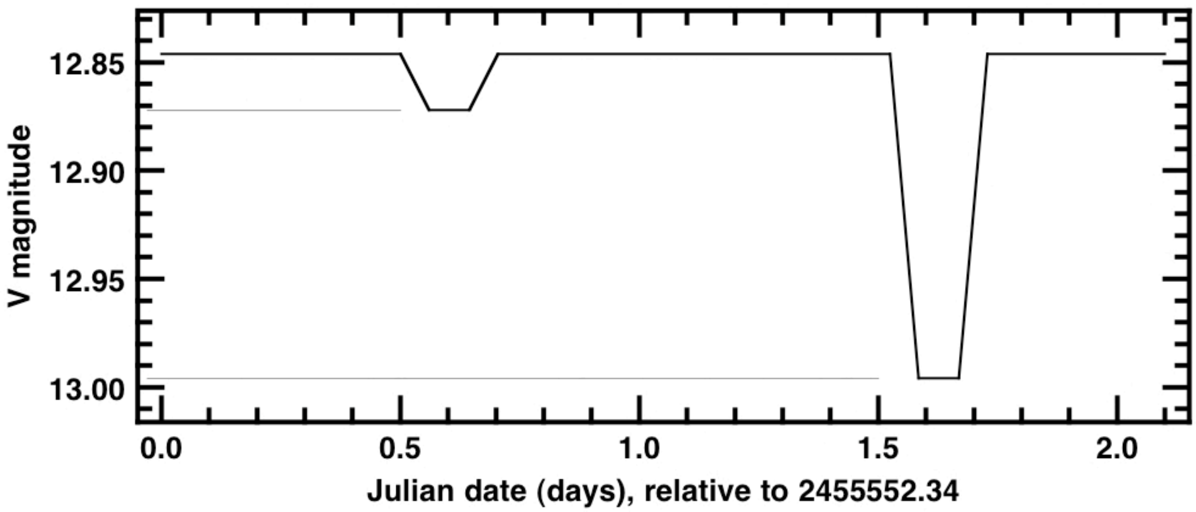
QUESTION 1. (Continued)   Major Figures in Astronomy

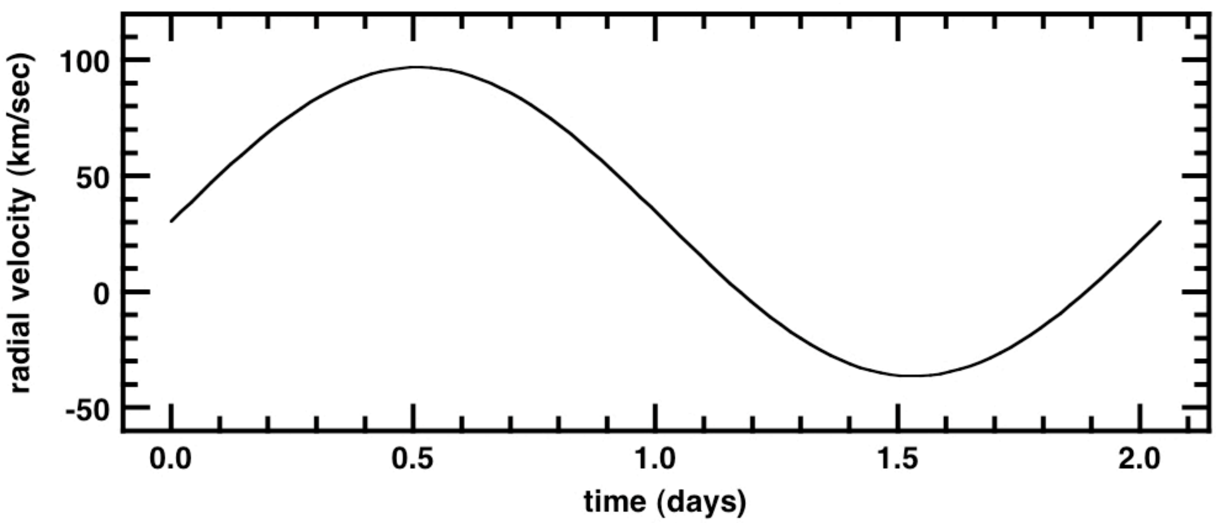
H-R Diagram for Part (a)   STUDENT NUMBER______________

(i) Draw and label the main sequence and other M-K luminosity classes.
(ii) On the x-axes, indicate: (i) all of the major spectral classes and (ii) 3-4 well-spaced values of the corresponding effective temperature.
(iii) Provide representative absolute magnitudes and luminosities on the relevant y-axis.
(iv) Draw in lines of constant radius for $100R_\odot$, $1R_\odot$, and $0.01R_\odot$.



REQUIRED: QUESTION 2. Binary Stars





A medium resolution spectrogram with a moderate signal to noise ratio identifies a particular star's spectral type to be A0V. Follow-up precision photometric monitoring of the star reveals that it is actually a binary with central eclipses, that has the light curve shown above.

a) Find the V magnitudes of each component of the binary system. (Tag the bright one as component 1, and the faint one as component 2. Note that the horizontal lines in the light curve diagram are there to assist the reader with this task.) [30 pts]

b) Assuming both stars have similar bolometric corrections, find **both** the ratio of the luminosities **and** the ratio of the masses of the two components. [30 pts]

c) Spectroscopic monitoring using higher quality spectrograms help produce the radial velocity diagram shown above, but continue to show the spectrum of only one star. Find the masses of both components, in units of solar masses. [30 pts]

d) What is the approximate spectral type of the secondary? [10 pts]

QUESTION 3. Time Scales for Stellar Processes

Stellar evolution is governed in its various phases by processes that have well defined time scales. This question asks you to describe some of these times, to give examples of their applications or consequences, and to offer ballpark values of these times for the Sun.

a) Name and describe (in words) the three fundamental time scales on which stars evolve. Give numerical values of these three time scales for the Sun. Also describe one phase of evolution of the Sun that is governed by each of these time scales. [45 pts]

b) Choose any ONE of the time scales from part (a) and derive (or explain) an accurate or precise formula for the time scale. Explain all your assumptions and approximations. Explain all terms that appear in your formula. [30 pts]

c) What is meant by the "convective time" (or convective turnover time)? Explain (in words) how you would go about estimating it, but do not try to carry out an estimate. Give what you think would be an approximate value or range of values for this time in the Sun's convection zone. Compare this time to the other times cited in part (a). What important consequence(s) does the value of this time have for the physical state of a convection zone in the Sun and in stars generally? [25 pts]

QUESTION 4. Initial Mass Function

Consider the Salpeter Initial Mass Function (IMF) of the form $\xi(M) = \xi_0 M^{-2.35}$ where ξ_0 sets the local stellar density.

a) Integrate the Salpeter mass function between a lower mass limit M_l and upper limit $M_u >> M_l$ to

  i) Find an expression for the number of stars formed. [15 pts]
  ii) Find an expression for their total mass. [15 pts]

b) Using reasonable assumptions about the relationship between stellar mass and luminosity for main sequence stars

  i) Find an expression for an equivalent stellar luminosity function between a lower luminosity limit L_l and upper limit $L_u >> L_l$. [15 pts]
  ii) Find an expression for the total luminosity. [15 pts]

c) Using your results from (a) and (b) and reasonable assumptions about upper and lower limits and the local stellar density,

  i) Calculate the integrated, bolometric, mass-to-light ratio for a young star cluster. [10 pts]
  ii) To what accuracy do you believe you have derived the mass-to-light ratio and what are the significant parameters that could effect the derived value? [10 pts]
  iii) How does your calculated mass-to-light ratio compare with the expected mass-to-light ratios for the Sun, an O-star, or an M-dwarf? (Explicitly calculate the mass-to-light ratio for these objects as part of your answer) [10 pts]
  iv) Briefly describe how you expect the mass-to-light ratio for this star cluster to change over time and why. [10 pts]

QUESTION 5. Gaseous Nebulae

a) Define and describe the **nebular approximation** as it relates to ionized nebulae. Be sure to include a justification for why the approximation is valid. [20 pts]

b) Define and describe the **on-the-spot approximation** as it relates to ionized nebulae. Be sure to include a justification for why the approximation is valid. [20 pts]

c) Consider a pure hydrogen nebula and assume ionization equilibrium applies. Derive a relation for the ionization fraction $f = n(H^+)/n_{tot}$. Be sure to define all symbols used. [30 pts]

d) Assume that the radiation field from the ionizing star is given by

$$J_\nu = 10^{-18}\left[\frac{\nu_0}{\nu}\right]^4 \text{ [erg s}^{-1}\text{ cm}^{-2}\text{ Hz}^{-1}\text{ ster}^{-1}\text{]},$$

where $h\nu_0$ = 13.6 eV, that the photoionization cross section is given by

$$a_\nu = 10^{-17}\left[\frac{\nu_0}{\nu}\right]^3 \text{ [cm}^2\text{]},$$

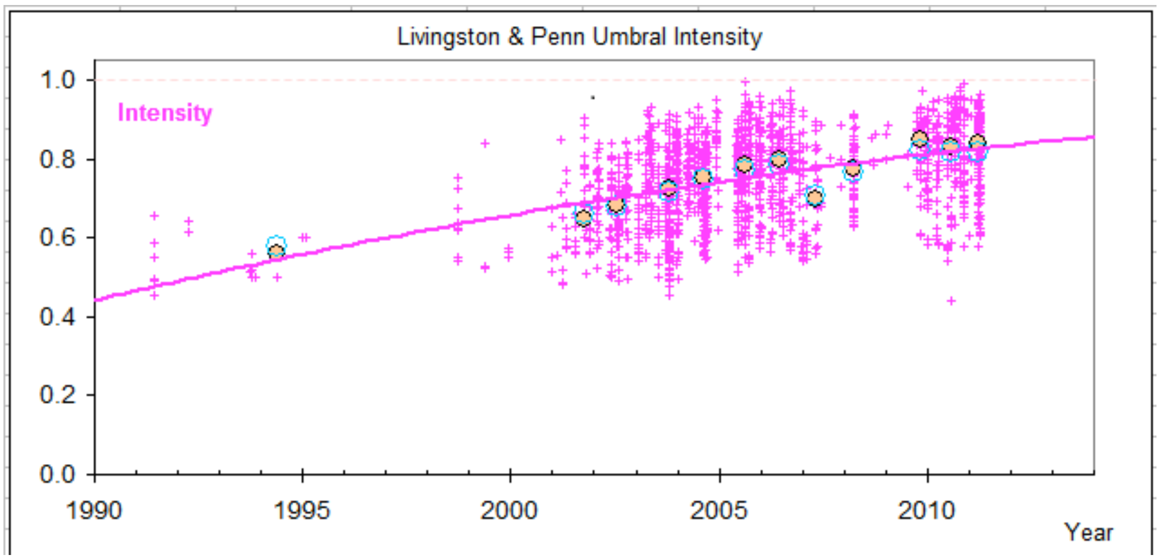
and that the relevant recombination rate is

$$\alpha_B = 2.6\times10^{-13}\left[\frac{10^4}{T_e}\right]^{1/2} \text{ [cm}^3\text{ s}^{-1}\text{]}.$$

Adopting these first-order parameterizations for the various quantities relevant to your answer to (c), compute the ionization fraction for the following case: T_e = 8000 K, and n_e = 200 cm^{-3}. [30 pts]

QUESTION 6. Sunspots

Livingston and Penn have measured the intensity of sunspot umbra for the last twenty years. Their data are plotted below.



a) From the graph above, estimate the approximate average temperature of sunspot umbra in 1990 and in 2010. Note that an intensity value of 1.0 corresponds to the photospheric intensity of the quiescent solar photosphere. Show your work and state any assumptions. [70 pts]

b) Describe how you would explain to the public the origin of the sunspot cycle, changes from cycle to cycle, and how the current sunspot cycle differs from previous cycles. [30 pts]

QUESTION 7. Dynamics

Planetary systems and galaxies present very different contexts for the variation of dynamical time with radius. Address these differences and their physical implications in the following questions using simple dimensional arguments and Newton's laws. Show all work.

a) Starting with Newton's laws, derive the form of M(r) implied by a flat rotation curve. [15 pts]

b) Derive the relationship between the orbital period and radius for a spiral galaxy in the flat rotation curve region. [15 pts]

c) Starting with Netwon's laws, derive the relationship between the orbital period and radius for a planetary system around a single star. Why is this result different than part (b)? [20 pts]

d) The radial distribution of light in the disks of spiral galaxies typically goes as $\exp(-r/r_d)$. What does r_d represent? If this disk was the only mass component in a galaxy, derive an expression for M(r), the mass enclosed as a function of radius, under the assumption that the mass to light ratio is constant everywhere within the galaxy. [30 pts]

e) Why do the answers to parts (a) and (d) present a problem? Briefly discuss a resolution that has been proposed to address this problem. [20 pts]

QUESTION 8. Telescopes

a) The "light-gathering element" of the typical dark-adapted human eye has a diameter of approximately 5 mm. How much fainter an object can be detected using the following telescopes with the specified primary mirror size? Write your answers both in terms of a number and a magnitude (for example, "the telescope can detect an object N times fainter and N magnitudes fainter than the eye"). [40 pts]

i) the WIYN 0.9m telescope
ii) the WIYN 3.5m telescope
iii) the Subaru 8.2m telescope
iv) the Thirty Meter Telescope (30m primary; currently being planned)

b) Every telescope has an intrinsic limit to the resolution it can achieve – the telescope's "diffraction-limited" resolution. Clearly and thoroughly explain what an Airy disk is and how it is related to the diffraction limit of a telescope. Use equations, and/or illustrations to supplement your written explanation if you wish. [20 pts]

c) Calculate the diffraction-limited resolutions for the WIYN 3.5m telescope and the Thirty Meter Telescope for observations in the Johnson V-band. [24 pts]

d) Compare your answers in part (c) to your estimate of the typical "seeing" achieved at a good astronomical site on Earth. Would you expect to obtain diffraction-limited images with the telescopes mentioned in part (c), without the aid of adaptive optics? Explain your answer. [16 pts]

QUESTION 9. Cosmology

a) In a universe that is expanding adiabatically (i.e., the Stefan-Boltzmann law holds), show that

$$(T/T_0) = (R_0/R),$$

where T is the temperature of the CMBR, R is the "scale factor" of the Universe and "0" = "today". [25 pts]

b) Use the relationship between the scale factor and the redshift $(R_0/R) = (1+z)$ to calculate the redshift of recombination. What is the age of the Universe when recombination occurs? [25 pts]

c) The figure on the next page shows the "Concordance Model" for Big Bang Nucleosynthesis. What is the significance of this figure? This figure gives a consistent value of $\Omega_m^0 = 0.24$ for the BBN production of the light elements. How does this figure require Dark Matter to be composed of particles different from ordinary protons, neutrons, and electrons? Explain. [25 pts]

d) What does the value of Ω_{total} imply about the geometry of the Universe? Cite and explain evidence that reinforces this conclusion. [25 pts]

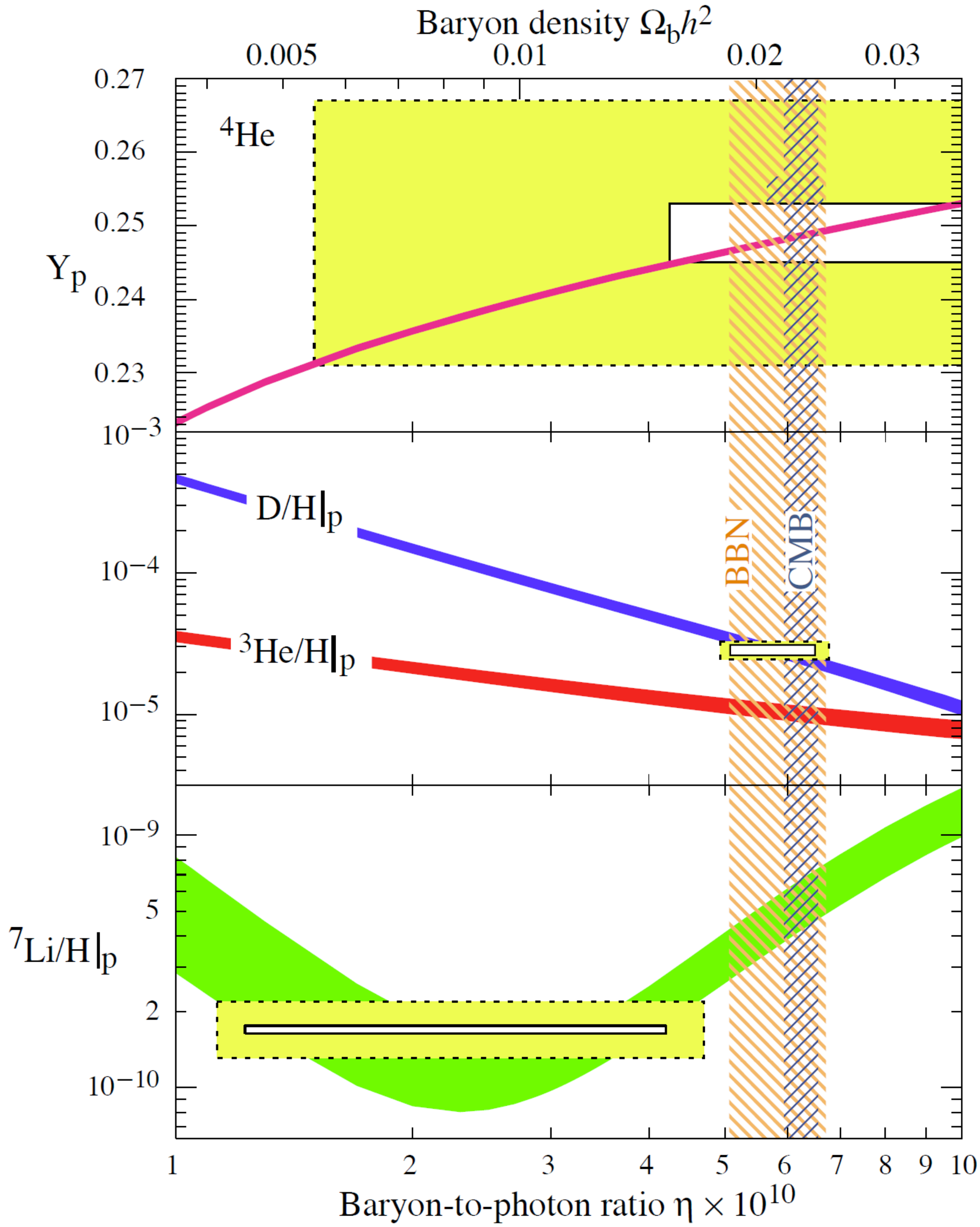


Figure 20.1: The abundances of ^{4}He, D, ^{3}He, and ^{7}Li as predicted by the standard model of Big-Bang nucleosynthesis [11] – the bands show the 95% CL range. Boxes indicate the observed light element abundances (smaller boxes: $\pm 2\sigma$ statistical errors; larger boxes: $\pm 2\sigma$ statistical *and* systematic errors). The narrow vertical band indicates the CMB measure of the cosmic baryon density, while the wider band indicates the BBN concordance range (both at 95% CL).

QUESTION 10. Public Policy

a) What is a “decadal survey” in astronomy and astrophysics? [20 pts]

b) How is the decadal survey carried out? [20 pts]

c) How are the recommendations of the decadal survey used, and by whom? [20 pts]

d) Identify the key science goals of four projects recommended by the most recent decadal survey in astronomy and astrophysics. [40 pts]