1. Mandatory Question: Distance Scales

Describe four methods that are used to determine distances in astronomy on galactic and extragalactic scales. For each method, explain the range of distances for which it is applicable and factors that limit its accuracy. (25 pts for each method)

2. Red Giant Interiors

a) Explain why the helium core of a $1\ M_\odot$ red giant does not undergo nuclear fusion until the star reaches the tip of the giant branch. What hydrogen-burning nuclear reactions occur on the red giant branch and where do they occur? Explain why and how the helium core evolves as the star ascends the giant branch. What role does electron degeneracy play? (40 pts)

b) Suppose that the average luminosity of a $1\ M_\odot$ red giant is $20\ L_\odot$. Also suppose that the mass of the helium core of the red giant increases $0.3\ M_\odot$ over the course of the red giant branch. Use this information to estimate the duration of the red giant phase in years.
(20 pts)

c) When helium fusion begins in a $1\ M_\odot$ red giant at the He flash, the luminosity of the core reaches an extraordinary high value, about $10^{11}\ L_\odot$, for a few seconds. Assuming that this phase lasts for 10 s, compute the energy released and compare this with an approximate binding energy for the core. Explain the significance of this result. Describe what happens next in the evolution of the star and why it happens.
(40 pts)

3. Star Cluster Evolution *

a) Consider a system of N stars of individual mass m that is in Virial Equilibrium with characteristic system radius R. Show that the strong scattering time t_{90} is of order Nt_{cr}, where t_{cr} is the crossing time. (30 pts)

b) Define the two-body relaxation time t_r and explain why it is *shorter* than the strong scattering time t_{90}. Recall that $t_r \sim t_{90}/(10 \ln N)$. (You do not need to derive this but discuss the origin of the $\ln N$ term.) Use this to compute a numerical estimate in years for the two-body relaxation time of a globular cluster with a total mass of $M = 2 \times 10^5 \, M_\odot$ and a half-mass radius of $r_h = 3 \, pc$. Discuss the physical significance of this result in terms of star cluster evolution. (40 pts)

c) Describe the expected dynamical evolution of a star cluster due to two-body relaxation, in the absence of binaries. Sketch the density profile on log-log axes at two-points in this evolution, indicating the sense of the evolution of the spatial structure. Discuss how this evolution is modified by the presence of hard binaries. (30 pts)

4. Galaxy Structure and Evolution

a) Discuss whether/how Hubble Type and star formation efficiency are linked. Your answer should include a discussion of the effects of environment and triggered star formation. (50 pts)

b) Discuss the role of feedback from either star formation or an AGN on the ISM. (50 pts)

5. CCD Detectors

Suppose you are using a CCD detector with the following characteristics: dark rate = 1.5 e^-/pixel/sec, readout noise = 5.0 e^-/pixel. On a typical dark (i.e. moonless) night at this particular observing site, the flux from the background sky yields a count rate of 7.5 e^-/pixel/sec. Answer the following questions clearly, using equations as well as complete sentences to explain your reasoning.

a) What is the noise level due to the three components mentioned above (dark, readout, sky background) in a 120-second exposure on a typical dark night? Explain the meaning of each of these noise sources (e.g., what type of noise is associated the dark level, where does the readout noise come from, etc...). (30 pts)

b) What is the total noise level due to the combination of these components? In other words, what is the total noise present in a 120-second image from these sources? Which noise source is the dominant component? Can any of these be ignored as an insignificant source of noise? Justify your answer. (30 pts)

c) Consider an image of a star taken with this CCD. You wish to measure the star's brightness. In addition to the three noise components mentioned above, what other noise term (if any) contributes to the uncertainty in the measurement of the brightness of this star? Explain your answer. (10 pts)

d) Using the answers to the parts above, write a general equation for the total noise associated with the measurement of a star on a single CCD image when exposure time is t seconds. Clearly define and explain all of the terms in your equation. (20 pts)

e) Given your answer to part (d), describe one thing that is commonly done (in terms of CCD operations at typical observatories) to reduce the noise of one component in the equation. Clearly explain why this works in practice to reduce noise. (10 pts)

6. Pillars of the Big Bang Theory *

It is often said that there are three observational pillars to the hot big bang model of the universe—three key observed facts upon which our belief in the model rests. Name these three pillars, and write a description of each. (100 pts)

7. The Jeans Mass and the Critical Mass *

Consider a spherical interstellar cloud of ideal gas with uniform temperature, density, and magnetic field. Ignore all surface terms and assume that the bulk velocity is zero. For parts (b) and (c), you may ignore factors of order unity; in other words, you may use Dimensional Analysis.

a) Write down the Virial Theorem for these assumptions. What term do you set equal to zero for the special case of Virial Equilibrium? (15 pts)

b) By setting the magnetic energy term to zero for Virial Equilibrium, derive an expression for the Jeans Mass M_J in terms of the gravitational constant G, the mean molecular weight μ, the mass of a proton m_p, the Boltzmann constant k, the temperature T, and the mass density ρ. (30 pts)

c) By setting the total thermal energy term to zero for Virial Equilibrium, derive an expression for the Critical Mass M_{crit} in terms of G, ρ, and the magnetic field strength B. How does M_{crit} change with ρ when the magnetic flux is frozen into the cloud? Explain. (30 pts)

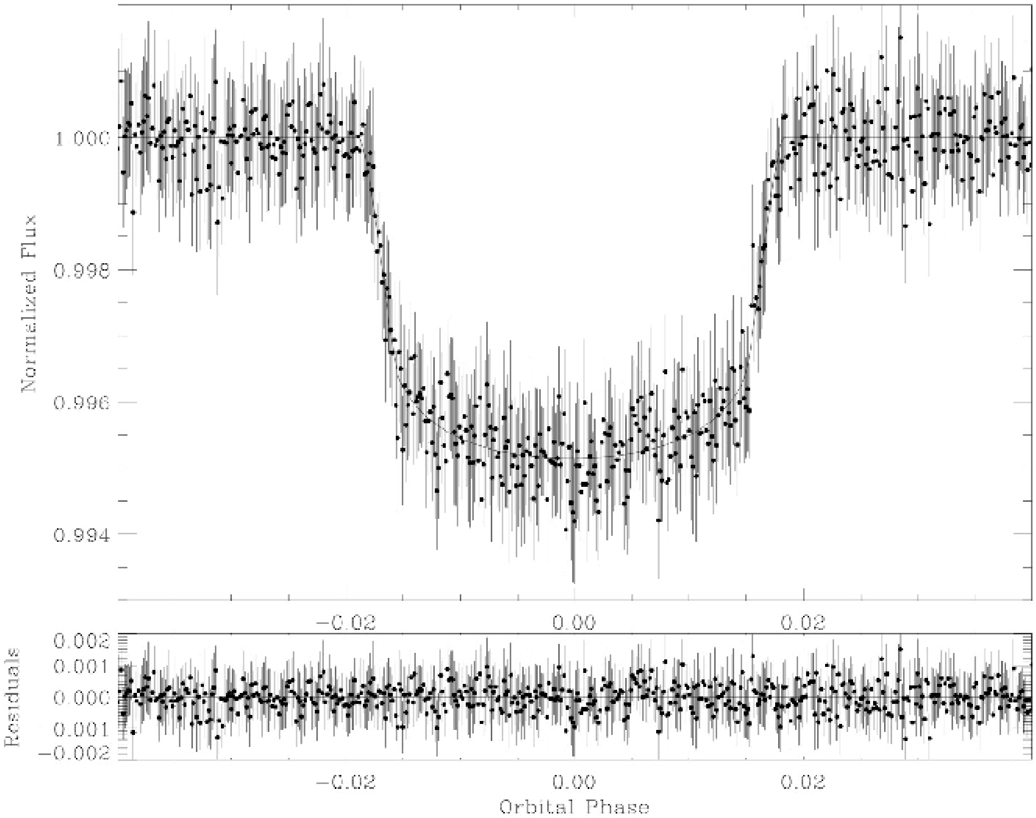
d) Cite typical values of ρ, T, μ, and B for Giant Molecular Clouds. For these values, what are typical values of M_J and M_{crit}. (Note: You do not have to plug numbers in the formulae, just cite typical values.) What is the significance of these results for star formation? (25 Pts)

8. Mystery Object

The ESA mission CoRoT has detected an object transiting the star CoRoT-Exo-3. The star itself is a spectral type F3V, with a mass of 1.37 solar masses and radius of 1.56 solar radii. The star has an effective temperature of 6740K and a surface gravity of 4.22. Its spectral lines are rotationally broadened at 17 km s^{-1}. Its distance is 680 pc.

Shown on the next page are the CoRoT data for the transit and the star's radial velocity curve, taken from Deleuil et al. 2008, A&A, 491, 889. The period of the transiting object is 4.26 days, and the eccentricity of its orbit is zero.

Estimate the semi-major axis of the object's orbit and estimate the mass, radius, and density of the transiting object. From these parameters determine what the object is. Show your work and describe the reasoning behind your conclusion. (100 pts)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0.998
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0.994
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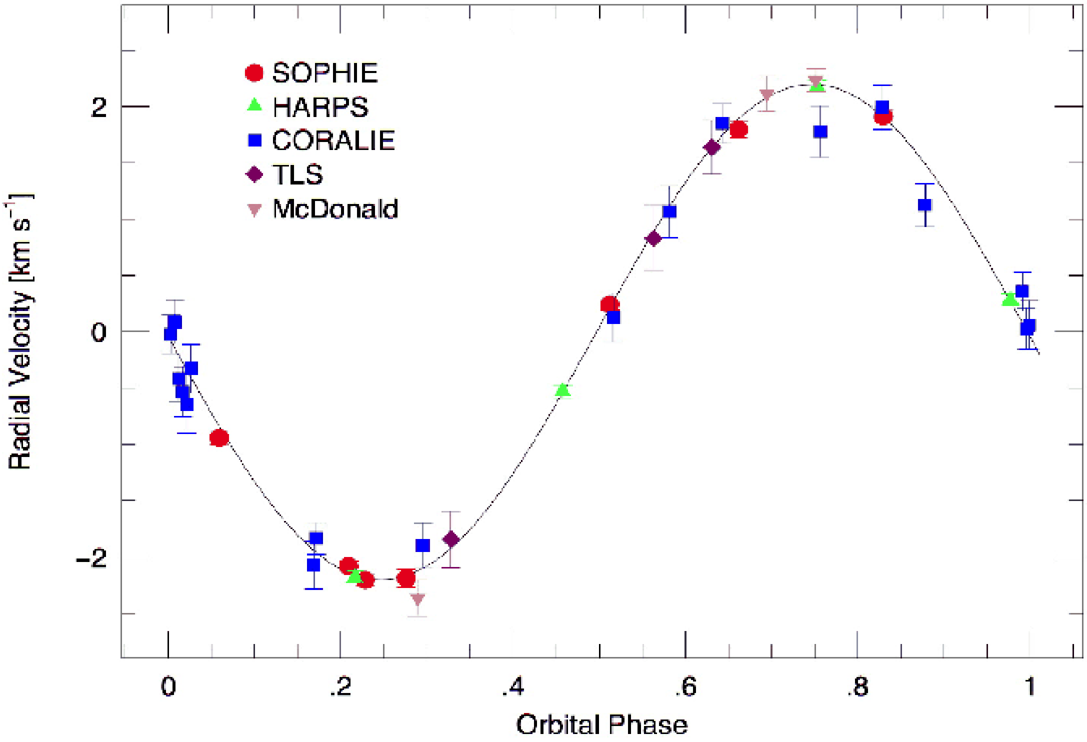
Orbital Phase


SOPHIE
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Radial Velocity [km s^{-1}]
2
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.6
.8
1
Orbital Phase

1. L_* Galaxies

Explain the concept of an L_* galaxy (provide a sketch if necessary). Is L_* invariant thoughout the history of the universe? Explain why or why not. Your answer should include a description of observational techniques that reliably identify galaxies at low and high redshift and also should include key words such as continuum, lines, star formation rate, metallicity, and density. (100 pts)

2. Main Sequence and White Dwarf Structure

a) For zero-age main sequence stars of low to moderate mass, the $T(\rho)$ structure in a log-log plot is relatively simple for the deep interior. Here T is temperature and ρ is mass density. Convective regions and radiative regions exhibit power-law behavior $T \sim \rho^a$ and $T \sim \rho^b$, where *a* and *b* are constants. What are the constants for these regions? Explain why they have these values. (30 pts)

b) When constructing a model for a deep stellar interior, how do you decide whether a region is convective or radiative? For simplicity, assume there is no composition gradient. (20 pts)

c) Explain why the gas in a white dwarf is electron degenerate rather than an ideal gas. Assume that complete nonrelativistic degeneracy applies. What polytrope does this correspond to? With this equation of state and hydrostatic equilibrium, find a mass-radius relation for low-mass white dwarfs, using Dimensional Analysis. (25 pts)

d) Explain why a mass limit is reached if we build a sequence of models of completely degenerate white dwarfs of increasing mass. What is the value of this limit? Discuss the effect of increasing white dwarf mass on the typical electron speed and on the equation of state as part of your answer. (25 pts)

3. Rotation Curves of Galaxies *

Rotation curves provide a fundamental tool for understanding the structure of galaxies, and have played a key role in developing the notion that dark matter is abundant in galaxies.

a) Draw an extended (to several radial scale-lengths) rotation curve observed for a typical isolated spiral galaxy. Label the axes and provide realistic axis scales. On the same figure, draw a dashed line depicting the shape of the rotation curve beyond the visible optical disk that was expected *prior to the time when it was possible to make observations of extended rotation curves.* How are these rotation curves observed at large distances from the centers of spiral galaxies? (25 pts)

b) Draw an extended (to several radial scale-lengths) rotation curve for an elliptical galaxy. Label the axes and provide realistic axis scales. How are these rotation curves observed?
(15 pts)

c) Explain how the observed rotation curves shown in parts (a) and (b) are related to the circular velocity at different radii. (20 pts)

d) Explain in detail why observations of extended rotation curves of spiral galaxies have led to the perceived need for "dark matter" in galaxies. Derive an expression for the enclosed mass inside a given radius, $M(r)$, consistent with the shape of these extended rotation curves at large radii. State any simplifying assumptions you make in this derivation. (20 pts)

e) Can the same approach be used with rotation curves of elliptical galaxies? Explain. Describe in detail a line of evidence indicating that elliptical galaxies possess dark matter halos.
(10 pts)

f) What fraction of the mass of galaxies is typically ascribed to dark matter? (10 pts)

4. The Early Universe *

a) Derive an expression for the variation of temperature as a function of time in the early universe when radiation dominates the energy density. Assume $\Lambda = 0$.
(25 pts)

b) Derive an expression of the variation of temperature as a function of time when matter dominates the energy density (k=0). Assume $\Lambda = 0$. (25 pts)

c) Derive an expression for the variation as a function of time if the cosmological constant dominates the energy density. Assume $\Lambda > 0$. This is how *Inflation* works in the early universe. (25 pts)

d) If $\Lambda > 0$, why must the universal expansion eventually become dominated by this Dark Energy? Schematically sketch the evolution of the universe in a $R(t)$ vs t diagram from the Big Bang until the universe becomes Dark Matter dominated. Where is our epoch on the evolutionary history? (Where are we now?) (25 pts)

5. The ISM in AGNs

Active Galactic Nuclei (AGNs) are observed to exhibit emission line spectra where both high and low ionization lines of various metal atoms are prominent. Examples of strong lines that are seen simultaneously in many AGN are [O I], [N II], [O III], and [Ne V].

a) Explain how it is possible for AGN to possess strong lines of both low- and high-ionization species (even of the same element, like oxygen). Hint: Part of your answer will need to include a discussion of the shape of the ionizing spectrum emitted by the AGN. (50 pts)

b) Given that we see strong high ionization features like He^{+2} recombination lines and Ne^{+4} forbidden lines, provide a quantitative lower limit to the energy range of the ionizing spectrum of AGNs. That is, what photon energies are required to deliver the observed ionization states? Give your answer in eV's, but also indicate the wavelengths corresponding to these energies. (25 pts)

c) Do we actually observe the ionizing photons of the type found in (b)? If YES, give examples of the observed spectra. If NO, then explain why. (25 pts)

6. Radiative Transfer in the ISM *

The transfer equation for the intensity of a monochromatic beam of radiation traversing a region in which the source function is S_ν can be written as:

$$\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu$$

where the optical depth τ_ν is defined in this case so that it *increases* in the direction that the radiation is moving. Assume constant S_ν for this problem.

a) Explain how the optical depth τ_ν is related to the mean free path ℓ_ν. (10 pts)

b) Show that the following is a solution of the transfer equation for constant S_ν. (30 pts)

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

c) Show using (b) that I_ν approaches the value of S_ν with increasing τ_ν. (10 pts)

d) Approximate an interstellar cloud as a uniform slab of optical depth τ_ν and uniform source function S_ν. Suppose the emission process is in local thermodynamic equilibrium (LTE) and the observation is being made in the radio at centimeter to meter wavelengths. Suppose that $I_\nu(0) = 0$. Show how you can rewrite the equation in part (b) in terms of brightness temperature T_b and the kinetic temperature of the gas T_{kin}. Explain your steps. (50 pts)

7. Stellar Radii

Two main sequence stars, A and B, have the following characteristics:

$V(A) = 9.5 = V(B);$

$E(B-V)$ towards $A = 0.50, E(B-V)$ towards $B = 0.00;$

$B.C.(A)$=3.2, $B.C.(B)$=1.0;

$D(A) = 2080\ pc, D(B) = 13.8\ pc$ (D is distance to Sun); and

$T_{eff}(A) = 28{,}000K, T_{eff}(B) = 3{,}480K.$

For the purposes of this question, assume the ratio of total to selective extinction is 3.

a) Find the ratio of the radius of star A to that of star B. (70 Pts)

b) What is the approximate spectral type of each star? (10 Pts)

c) Describe the spectrum of each star. (20 pts)