1. Mandatory Question: Time Scales for Stars

This problem will guide you through a series of questions where your perspective moves through history as you attempt to understand how long the Sun lives. Assume you always know Newton's theory of gravity, Newtonian dynamics, and the Sun's basic characteristics (such as luminosity, mass, radius, and composition). With each part of the question, you become aware of more physics that you can use to address that part of the question. In each case, give a numerical estimate for the associated time scale and explain your estimate.

a) Dynamic time scale. Estimate the dynamic or free-fall time for the Sun defined as how long it would take the Sun to collapse from its present size to negligible size in the absence of pressure support. Please offer at least a dimensional argument for whatever formula you use for this time scale. Evaluate your estimate. (30 pts)

b) Chemical time scale. By the late 18th and early 19th centuries, scientists knew how to quantify the amount of energy involved in chemical reactions and speculated that the Sun's luminosity was fueled by chemical reactions. Use your knowledge of a "typical" energy per atom that can be obtained from chemical reactions to estimate a chemical lifetime of the Sun. (20 pts)

c) Kelvin-Helmholtz time scale. Later in the 19th century, with a better understanding of thermal physics and the hydrostatic equilibrium of a star, it was thought that the Sun might be powered by the slow release of gravitational potential energy. Explain how this leads to the Kelvin-Helmholz time scale and estimate the KH lifetime of the Sun. (30 pts)

d) Nuclear time scale. Finally, jump to the physics world of the 1930's and 1940's, and assume that it is nuclear reactions that power the Sun. Assuming a "typical" energy per nucleon from relevant fusion reactions, estimate the nuclear lifetime of the Sun. (20 pts)

2. Earth to Mars

a) The minimum energy orbit that will take a spacecraft from Earth to Mars has its perihelion at Earth's orbit and its aphelion at Mars' orbit. Assuming that Earth and Mars have perfectly circular orbits and given that the semi-major axis of Mars' orbit is 1.52 AU, what are the semi-major axis a and eccentricity e of the spacecraft's orbit? How long does the spacecraft take to reach Mars? Sketch the orbits of Earth, Mars, and the spacecraft. (50 pts)

b) Derive an expression for the energy of a spacecraft of mass m in a circular orbit of radius a about the Sun, in terms of the solar mass $M_{\odot}$, m, a, and physical constants. Remembering that the energy of an elliptical orbit is given by the same expression with a as the semi-major axis, use your result to derive the vis-viva equation:

$$v^2 = G M_{\odot}(2/r - 1/a).$$

In this expression, v is the instantaneous orbital speed of the mass when the mass is at a distance r from the Sun. Now, assume the mass starts in Earth's orbit. Neglecting the gravitational force of Earth on the spacecraft, calculate the change in speed and direction *relative to Earth*, $\Delta\mathbf{v}$, with which the spacecraft must be launched in order to reach Mars. (50 pts)

3. Absorption Line Profiles

Consider thermal gas in stellar atmospheres and in cold interstellar gas clouds where the magnetic field and rotation of the stars or clouds are negligible. Answer the following questions about absorption lines produced by atoms or ions in the gas.

a) First focus on stellar atmospheres. Name and describe the principal line broadening mechanisms expected in stellar atmospheres, including a physical description of the mechanism and the kind of profile each mechanism tends to produce. (25 pts)

b) What is meant by the "curve of growth" of an absorption line? Name and explain the three distinct parts of the curve of growth and how the equivalent width varies with column density of the atom or ion as its abundance (column density) in the stellar atmosphere is increased. (25 pts)

c) For stars of the same spectral class, one way to distinguish the luminosity class of stars is to compare the widths of spectral lines. Explain the physical basis for this connection between line width and luminosity class. (15 pts)

d) Explain what the term LTE (local thermodynamic equilibrium) means as applied to a stellar spectral line. If a line is in LTE, explain why a strong absorption line in a stellar atmosphere is not completely black at the line center. (15 pts)

e) How do the principal broadening mechanisms for absorption lines produced by interstellar clouds differ from those formed in stellar atmospheres? How can you distinguish strong interstellar atomic absorption lines in a stellar spectrum from strong absorption lines produced in the stellar atmosphere just by their appearance? (20 pts)

4. Chemical Evolution of Galaxies

Choose one of the many different ways to trace chemical evolution in extragalactic systems. Describe the observations and analysis you would need to undertake an observational program to investigate chemical evolution. Take care to delineate clearly not only the type of observations, but also the wavelength coverage, the lines or continuum of interest, the required calibration, and the details of the analysis needed to achieve your scientific goal. (100 pts)

5. T Tauri Stars

a) Sketch both a P Cygni spectral line profile and an inverse P Cygni spectral line profile. For each, also sketch the physical source that can produce the line profile and explain how the line profile arises from the source. Which is associated with mass accretion and which with mass loss from T Tauri stars? Which type of profile is observed in T Tauri stars? (20 pts)

b) Typical mass loss rates from T Tauri stars are 10^{-7} solar masses per year, with stellar wind velocities of around 80 km s^{-1}. Estimate the mass density of the wind at 100 AU from the star. (20 pts)

c) Describe at least three lines of evidence that suggest that T Tauri stars are young and still evolving toward the main sequence, and not older stars evolving off the main sequence? (20 pts)

d) Define the following and describe their origin: (20 pts)

- Herbig-Haro object
- FU Orionis star
- Proplyd

e) Conservation of angular momentum would suggest that stars arriving on the main sequence should be rotating near break-up velocity. They are not. How is the loss of angular momentum explained, and what evidence supports this hypothesis? (20 pts)

6. CCD Imaging

Imagine that you are operating a CCD that is attached to the back of a telescope and that is being used to image the sky. Answer the following questions about the operation and properties of the CCD. The use of diagrams is encouraged.

a) Explain in detail how the CCD works – how does the CCD "capture" and "store" the incoming photons? (25 pts)

b) How is the information that is contained in the CCD read out to form a coherent image once the exposure is finished? (25 pts)

c) List and thoroughly discuss the advantages and disadvantages of CCDs as astronomical detectors. (35 pts)

d) Most research-grade CCDs are sensitive to photons in the range 4000 Angstroms to 1.1 micron. Explain what steps can be taken to increase the quantum efficiencies of CCDs at the ultraviolet and infrared ends of the response curve. (15 pts)

7. Dark Energy

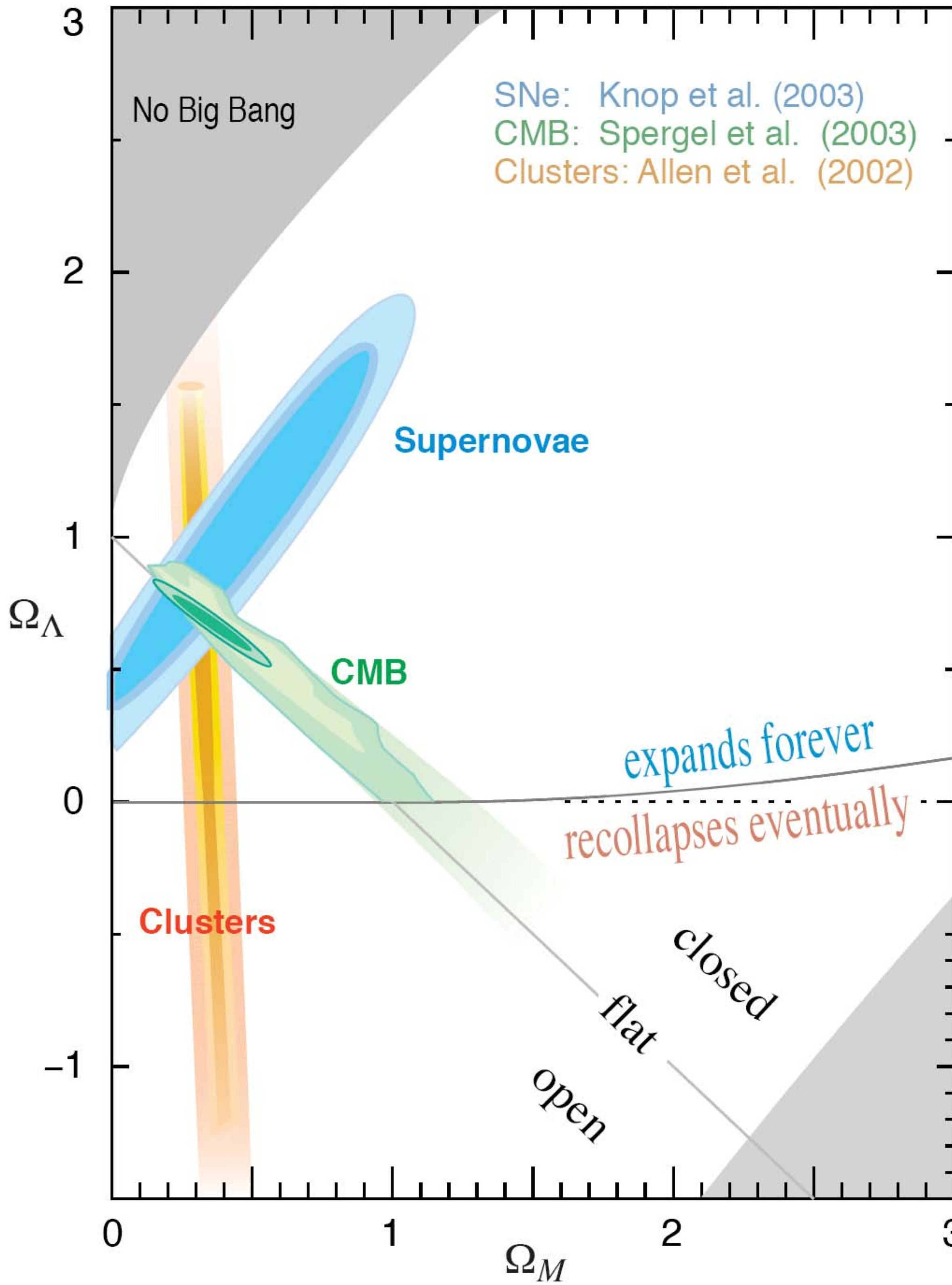
Refer to the Figure on the following page for these questions.

a) What measurements are used to construct the error ellipses labeled “Supernovae”? Be specific in describing the measurements and what measure of distance is used in interpreting the measurements. How do these measurements determine Ω_Λ and Ω_M? (25 pts)

b) What measurements are used to construct the error ellipses labeled “CMB”? Be specific in describing the measurements and what measure of distance is used in interpreting the measurements. How do these measurements determine Ω_Λ and Ω_M? (25 pts)

c) What are the fractions of ordinary matter, dark matter, and relativistic matter that make up Ω_T in the Universe today? What fraction of ordinary matter do we currently “see” as visible matter? What kind of particle is the dark matter thought to be? (25 pts)

d) Describe the overall conclusions that can be drawn from this Figure. (25 pts)



No Big Bang
SNe: Knop et al. (2003)
CMB: Spergel et al. (2003)
Clusters: Allen et al. (2002)
Supernovae
CMB
Clusters
expands forever
recollapses eventually
closed
flat
open
Ω_Λ
Ω_M
3
2
1
0
−1
0
1
2
3

8. Quasars and the Lyα Forest

a) What are quasars (a.k.a. QSOs)? In approximate numbers, give estimates for the luminosity characteristic of quasars (in solar units or absolute magnitude) and the range of distances (in Mpc) where they are found. Describe their appearance, both in images and spectra, as seen by astronomers on earth. What is the current best model to explain their energy output? (40 pts)

b) Spectra of distant quasars often show hundreds of weak, narrow absorption lines blueward of the Ly-α emission line from the quasar. These are referred to as "Lyman-α forest lines". What is causing this absorption? What can we learn by studying these absorption lines? (40 pts)

c) Why do we not see any Lyman-α forest lines redward of the Ly-α line emitted by the quasar? (20 pts)

1. The Evolution of the Sun

Describe the evolution of a one solar mass star over its entire lifetime, starting with its pre-main-sequence contraction on the Hayashi track. Discuss each evolutionary stage, including details about what is occurring in the star's core and envelope such as primary energy source(s), modes of energy transport, and equation of state. Give estimates for the amount of time spent in each stage, as well as for the entire lifetime of the star. Supplement your written description by showing the star's evolution on an HR diagram and on a central temperature T_c versus central density ρ_c plot. Label the axes of these diagrams carefully. Show the full main sequence on the HR diagram and show the hydrogen, helium, and carbon ignition curves plus the dividing line between degenerate and nondegenerate gas on the T_c vs ρ_c plot. Use letters to relate positions on the HR diagram evolutionary track with positions on the T_c vs ρ_c evolutionary track of the star. (100 pts)

2. Gravitational Accretion onto Stars

a) Suppose that a mass *m* falls radially from rest at a large distance onto the surface of a collapsed star of mass *M* and radius *R* and is stopped by a shock, where it releases its kinetic energy. Compute the fraction of the rest energy that is released in terms of *M, R*, and physical constants. Evaluate your expression for the cases of: (1) a 1 $M_{\odot}$ white dwarf and (2) a 1.4 $M_{\odot}$ neutron star. Assume that non-relativistic gravity applies in both cases and use appropriate radius values. Compare your results for each case to the efficiency of hydrogen fusion for converting mass to energy. (40 pts)

b) Suppose that instead of falling radially, the mass spirals into the surface of a collapsed star through a Keplerian accretion disk that extends inward to the surface of the star. Show in this case that the kinetic energy of the material reaching the star is only half as large as in the radial infall case. Account for this difference. (30 pts)

c) Define the Eddington luminosity limit L_E for energy release by accretion of ionized gas onto a collapsed object (like a white dwarf or neutron star). Derive an expression for L_E in terms of basic physical constants and the mass of the collapsed object *M*, assuming the dominant opacity source is electron scattering. (30 pts)

3. Flat Rotation Curves

Since the late 1960s "extended" flat rotation curves have been obtained for a large number of spiral galaxies. These observations have led to major changes in how we view galaxies.

a) Draw an extended flat rotation curve, with axes labeled and with realistic axis scales, for a typical isolated spiral galaxy. Explain how the rotation curve is related to the circular velocity at different radii. In what ways do rotation curves observed in elliptical galaxies differ from this? (30 pts)

b) How are rotation curves observed at large distances from the centers of spiral galaxies? At the quantum level, describe the transition associated with these observations. What type of transition is this? In what phase of the ISM does this originate? What are the physical characteristics of this phase? How are rotation curves of elliptical galaxies observed? (30 pts)

c) Explain in detail why extended rotation curves have led to the perceived need for "dark matter" in galaxies. Derive an expression for the enclosed mass inside a given radius, M(r), consistent with the shape of extended rotation curves at large radii (you may assume here that the mass has a spherical distribution). Describe in detail two additional lines of evidence indicating that dark matter exists in the universe. What fraction of the mass of galaxies is typically ascribed to dark matter? (25 pts)

d) Alternatives to dark matter have been proposed to account for the extended flat rotation curves. These alternatives all posit that there is no "unseen" matter, i.e., the mass distribution follows the light distribution, and have in common a modification of Newtonian dynamics in regimes where the gravitational acceleration is very small. The best known of these, MOND (**MO**dified **N**ewtonian **D**ynamics), proposes that when the gravitational acceleration $\boldsymbol{g}$ from an external mass distribution falls below a very small, but nonzero, critical value $\boldsymbol{a}_{crit}$, the gravitational potential is no longer Newtonian but rather takes a form that yields a constant circular velocity at increasingly larger distances from the mass distribution. Derive the form of the gravitational potential $\Phi(r)$ that yields $v(r)$ = *constant.* (15 pts)

4. HII Regions

a) Suppose you took a spectrum of an HII region like the Orion Nebula. Describe what it would look like. Be as detailed as possible, and assume that the spectrum covers the optical portion of the EM spectrum. What spectral features are most prominent? (25 pts)

b) HII regions are usually described in terms of a steady-state configuration that is characterized by ionization equilibrium and thermal balance. What is the single most important source of input energy to the nebula (be as specific as you can)? Describe the evidence that supports your answer. Into what forms does this energy get transformed after being absorbed by the gas? (25 pts)

c) What are the primary ways that the nebula loses energy, and what are the observational signatures of each of the loss mechanisms? Your answers should include a brief description of the physical processes involved. (30 pts)

d) How would your answer to (c) change if the nebula is pure H gas with no He or metals? Would the nebula likely be hotter or cooler in this case? (20 pts)

5. The Cataclysmic Variable Z Cham

The eclipsing dwarf nova Z Chamaeleontis consists of a white dwarf primary of 0.85 solar masses, a radius of 0.01 solar radii, and a temperature of about 16,000K, and a late-M main sequence star with a mass of 0.17 solar masses. The orbital period is P=0.0745 days. The system is at a distance of about 125 pc.

a) Calculate the separation of the two stars in solar radii and in centimeters. Show your work. (30 pts)

b) The radius of a 0.17 solar mass M dwarf is approximately 0.38 solar radii. Calculate the distance of the inner Lagrangian point L_1 from the secondary star, and compare this distance with the approximate radius of the secondary. Is the Z Cha system detached, semidetached, or a contact binary? (The separation between the two stars is about 5.2×10^{10} cm). Show your work. (30 pts)

c) The mass transfer rate onto the accretion disk during an outburst is approximately 1.3×10^{-9} solar masses per year, and the maximum disk temperature is roughly 44,000K. Describe a very rough "light budget" of the system, indicating the relative contribution to the total system luminosity and the peak wavelength of emission of each component of the system (primary, secondary, and accretion disk, if present). Be sure to include what assumptions you make about the primary, secondary, and possible accretion disk components. (40 pts)

6. Statistics

Answer the following questions about basic statistics. Be clear and complete in your answer. Use examples from astronomy/astrophysics, illustrations, or diagrams as needed.

a) What is the difference between precision and accuracy? (10 pts)

b) What are systematic errors? What are random errors? Discuss how random errors can be reduced in an experiment or series of experiments. (20 pts)

c) What is a Poisson distribution? To what types of experiments/situations does it apply? Draw an example Poisson distribution and label (approximately) the mean, standard deviation, and mode of the distribution. Write an expression for the standard deviation as it relates to the mean value of the Poisson distribution. Finally, give a specific example from astronomy to which a Poisson distribution would apply. (35 pts)

d) What is a Gaussian distribution? To what types of experiments/situations does it apply? Draw an example Gaussian distribution and label (approximately) the mean, standard deviation ("σ"), mode, median, and full width half maximum (FWHM) of the distribution. Mark and label the locations of the 1σ, 2σ, and 3σ distances from the mean of the distribution, and explain their significance. Finally, give a specific example from astronomy in which a Gaussian distribution would apply. (35 pts)

7. Cosmic Microwave Background Radiation

Refer to the Figure below for these questions.

a) The CMBR is almost a perfect black body. What does this imply for the physical conditions in the early Universe? Explain. (20 pts)

b) Explain the axes in this figure. What is actually being measured? What introduces temperature fluctuations into the CMBR? (20 pts)

c) The l-pole of the first "Acoustic Peak" determines the geometry of the Universe by using the sound horizon as a metric ruler. What is the sound horizon? Derive an expression for the radius of the sound horizon at the time of recombination - that is, solve Friedmann's equation in the appropriate limit for $R(t_{rec})$ and evaluate any integral(s). (35 pts)

d) Derive an expression for the diameter of the sound horizon at recombination as it is "today". Derive an expression for the angular diameter of the sound horizon as seen on the sky today. (25 pts)