PH.D. QUALIFYING EXAMINATION

Department of Astronomy

August 15, 2007

1:00pm—5:00pm

DAY ONE

Name: ______________________________

Student #: ___________________________

The exam sheets are inside this envelope and are not fastened together. Astronomy Program students **MUST** do the FIRST problem and SIX more problems from the remaining **SEVEN**. Astrophysics Program students **MUST** do the **FIRST** problem and **THREE** or **FOUR** more problems from the remaining **SEVEN**. When you are finished, please put the questions and your answer sheets back in the envelope in the correct order. Be sure the student number given you by the proctor is on every page of your answers.

Physical Constants

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$h = 6.63 \times 10^{-27}$ erg s

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$m_p = 1.67 \times 10^{-24}$ g

$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}

$e = 4.80 \times 10^{-10}$ esu

$m_e = 9.11 \times 10^{-28}$ g

$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}

$\sigma_T = 6.65 \times 10^{-25}$ cm^2

$R_\odot = 6.96 \times 10^{10}$ cm

$M_\odot = 1.99 \times 10^{33}$ g

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A.U. $= 1.50 \times 10^{13}$ cm

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$M_{V\odot} = 4.83$ mag

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1. Hertzsprung-Russell Diagram

   a.) Use graph paper to construct a theoretical HR-diagram log L versus log T_{eff}. Assume that the diagram will cover the full range of properties for normal stars (excluding neutron stars and black holes). (75 pts)

      i. Next to the tick marks on the left axis, indicate values for the luminosity L. Be sure to span the full range of stellar luminosities for normal stars. On the right axis show tick marks for M_{bol} making sure these are self-consistent with the values on the left axis. Draw three horizontal axes. Label one "T_{eff}", and mark off 50,000 K, 10,000 K, 6,000 K, and 3,000 K. Label another "spectral class" and write in all the relevant spectral classes. Label the third "B - V", and fill in the appropriate numbers. Be sure your answers for these bottom axes are roughly self-consistent with each other.

      ii. Draw in the Sun by placing a "s" symbol at the correct location.

      iii. For Population I stars, sketch in and label the following regions: main sequence, giants, supergiants, and white dwarfs.

      iv. Draw in lines of constant radius, for 10^{-2}, 1, and 10^{2} $R_{\odot}$.

   b.) Sketch in the pre – Main Sequence evolutionary tracks for 1 and $10M_{\odot}$ stars and the evolutionary track of a brown dwarf. (25 pts)

2. Galaxy Morphology

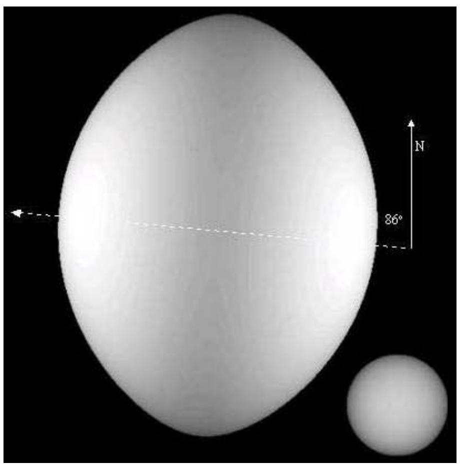
a.) Describe how S0, Sa, and Irr galaxies differ in surface brightness, current star formation activity, linear size, and global color (B-V). Include a sketch of their surface brightness profiles as a function of radius (in the B-band). Discuss how their stellar populations differ and sketch the approximate global star formation rate as a function of time for each galaxy type. (50 pts)

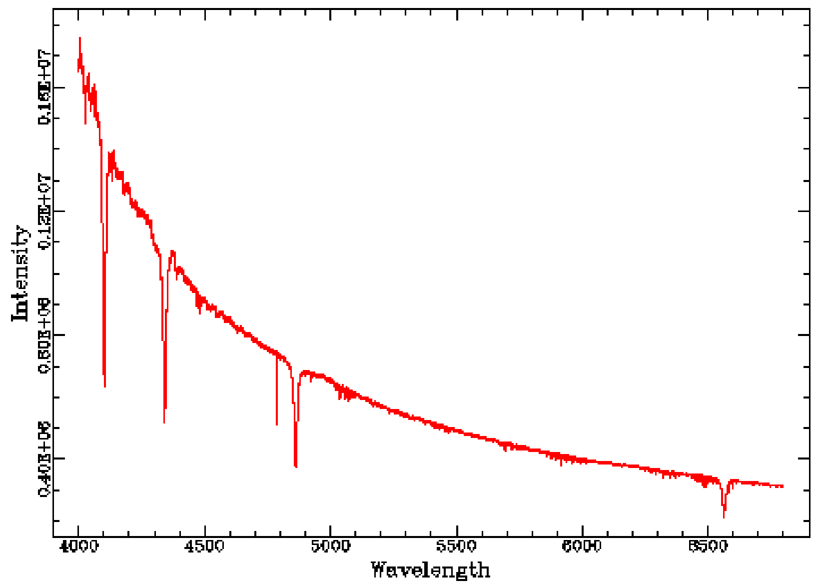
b.) Discuss the role environment plays in galaxy formation and evolution. (Galaxy clustering, large-scale structure, galaxy interactions, and the morphology-density relation might be considered.) (25 pts)

c.) Describe the typical environment in which you might find a large population of dEs. Describe the typical environment in which you might find a large population of dIs. What is the observational evidence for or against dIs and dEs being evolutionarily-linked? (25 pts)

3. Stellar Rotation and Spectral Characteristics.

An image of the star Regulus, derived from interferometric measurements, shows the distortion of Regulus due to its rapid rotation. Regulus is rotating around an axis in the plane of the image, shown by an arrow in the figure. The Sun is shown for comparison in the lower left corner. Regulus is spinning at 86% of its breakup speed. The temperature at the poles of the star is about 15,000 K and the temperature around the equator is about 10,000 K. The mass of Regulus is about 3.5 solar masses. Its spectral type is B7V. A spectrum of Regulus from the ELODIE spectral archive is shown below.





a.) Identify the principal features in the spectrum. (10 pts)

b.) Calculate the surface gravity (expressed as log g) of the poles and the equator. (30 pts)

c.) Estimate the spectral types (not subtypes!) and luminosity types of the poles of Regulus and of its equatorial regions. Explain your answers in detail. (30 pts)

d.) Describe how the optical and ultraviolent spectrum would change if we had a pole-on view of Regulus rather than an equator-on view. Explain the physical reasoning behind the changes you would expect. (30 pts)

4. An Earth-like Planet?

The detection of a possibly Earth-like planet orbiting the M3V star Gliese 581 has recently been reported, based on radial velocity measurements. The orbital parameters of the planet Gliese 581c are P = 12.9 days and a = 0.073 AU and the mass of the planet has been estimated at $M_C \sin i = 0.016\ M_J$ where M_J is Jupiter's mass and i is the unknown inclination of the planet's orbit. (Recall that $M_J \approx 10^{-3}\ M_\odot$.)

a.) Compute the mass of the star from the orbit of this planet. Is your result consistent with a star of this type? (30 pts)

b.) This planet was detected using the radial velocity method, from the motion of its star. Estimate the amplitude of this stellar motion. You may assume a circular orbit. What observational technique is needed to measure a motion of this amplitude? (35 pts)

c.) Compare your result from (b) with the amplitude of the motion of the Sun due to the Earth's orbit about it. Discuss the significance of this result for the detection of a true Earth "twin." (20 pts)

d.) While Gliese 581c has been proposed to be a rocky object, it is alternatively possible that it is an icy object. What type of observations might help to distinguish between these possibilities? (15 pts)

5. Collisions in the Galactic Center

a.) Discuss two lines of evidence that support the conclusion that the nucleus of the Milky Way Galaxy contains a supermassive central black hole. (30 pts)

b.) The mass of the central black hole in the Milky Way Galaxy has been determined to be $M_{BH} = 3.7 \times 10^6 M_\odot$. The stellar mass density distribution surrounding the black hole may be approximated by $\rho_s(r) = \rho_0 \left(r/r_0\right)^{-\alpha}$, with $\alpha = 1.8$. Show that the total stellar mass within r_0 is finite and determine ρ_0 for r_0 = 1 pc and $M(r_0)$ = 1.4 x 10^6 $M_\odot$. How does ρ_0 compare with the stellar mass density in the solar neighborhood? (30 pts)

c.) Find the collision time for a red giant as a function of r, and determine the value of r for which the collision time is 10^9 yr. Use a simple "$n\Sigma v$" calculation and take the red giant radius to be $R_{RG} = 10 R_\odot$. Assume that the red giant is in a circular orbit and that the rotation curve is determined by the black hole alone. Also assume that the stellar distribution is dominated by solar type stars. (40 pts)

6. LTE in the ISM

a.) What is required in the ISM for an emission line to be in LTE? Your answer should include an explanation of what LTE means in this context. (30 pts)

b.) You observe an LTE emission line from a nebula in the radio. Suppose you know that the optical depth is 0.5 along the line of sight through the nebula and that the observed brightness temperature is 20K. What is the kinetic temperature? Explain your answer. (40 pts)

c.) Consider emission lines from rotational levels of ^{12}CO and ^{13}CO. Assume they are in LTE. Explain how observations for the same molecular cloud of these two molecules can be used to determine both the temperature T and number density n of the cloud. (30 pts)

7. "Cold" Astrophysical Objects

Consider the radius-mass (R-M) relation for spherical objects in hydrostatic equilibrium where the matter is cold (T = 0) and catalyzed, i.e., in its lowest energy state. In case you do not remember, for low to moderate densities, "catalyzed" matter will be ^{56}Fe. Although real astronomical objects are unlikely to be catalyzed in this sense, the R-M relation is useful for characterizing the various types of cold compact objects we expect to find in the Universe.

a.) Sketch the R-M relation for cold, catalyzed matter in log R versus log M. Indicate what real astrophysical objects lie near the various parts of the R-M curve (terrestrial planets, giant planets, white dwarf, etc.). Indicate approximate masses at special turning points in mass or radius with some accuracy. (50 pts)

b.) Show the parts of the curve where you expect the objects to behave like polytropes with effective polytropic indices of 0, 1, 3/2, 3, and > 3. (25 pts)

c.) One of the prominent features of your plot will be the maximum mass at the end of the sequence that represents the catalyzed analogues of white dwarf stars. Explain why this maximum mass occurs. (25 pts)

8. Friedmann's Equation

a.) Using the first two equations on the next page, derive Friedmann's Equation, a second order, ordinary differential equation for $R(t)$. (10 pts)

b.) In a radiation dominated universe (early universe), how does R scale with the temperature T? Using this variation, transform Friedmann's Equation into an equation that describes the time evolution of the temperature in a radiation dominated universe. If the temperature is very high, does geometry matter? Assume $\Lambda = 0$ and solve this equation for $T(t)$ in the early universe. (35 pts)

c.) In a matter dominated universe, how does the energy density scale with R? Using the scaling, derive an equation for the evolution of R as a function of time in a flat, matter dominated universe. Assume $\Lambda = 0$ and solve this equation. (35 pts)

d.) Suppose the universe is dominated by Dark Energy or Inflationary Energy. That is, the cosmological constant term $\Lambda > 0$ dominates the energy density terms and geometry terms in Friedmann's Equation. Derive an equation for the evolution of R as a function of time in these universes. Solve this equation. In case of Dark Energy, what is the fate of this universe? (20 pts)

PH.D. QUALIFYING EXAMINATION

Department of Astronomy

August 16, 2007

1:00pm—4:30pm

DAY TWO

Name: ______________________________

Student #: ___________________________

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1. Rotation Curves

   a.) Using simple dynamical arguments, derive the functional form for mass density ρ as a function of radius r if the galactic disk exhibits:

      i. A flat rotation curve. (25 pts)

      ii. Solid body rotation. (25 pts)

   In each case, indicate what regions of a spiral galaxy is best described. Also, discuss why neither of these two types of rotation curve can accurately describe an entire galaxy.

   b.) Describe and illustrate the maximum disk method of rotation curve fitting. Be sure to describe the origin of the data you would use as well as any underlying assumptions you must make in order to fit the distributions. (50 pts)

2. Multiple Stars.

The age of the B7V star Regulus (M_V = -0.5) is estimated to be about 5 x 10^7 years. Regulus is orbited by a companion at a distance of 4,200 AU. The companion is itself a binary star, containing a K5V (M_V = 4.2) and an M5V (M_V = 9.5) main sequence star. Their separation is 100 AU, and the distance from the Sun to the Regulus system is 24 pc.

a.) Estimate the angular separation of the M5 +K5 binary pair on the sky. What method would be needed resolve the two stars? (20 pts)

b.) What observations would be most useful to confirm that the binary pair is gravitationally bound to Regulus? (20 pts)

c.) Given the rapid rotation of Regulus ($v \sin i$ = 315 km s^{-1}) and the age of the system, what rotational velocities would you expect for the K5V and the M5V star? (20 pts)

d.) Describe the principal spectral features you would expect to see in the spectra of the K5V and M5V stars. (20 pts)

e.) Over the evolutionary lifetimes of the K5V and the M5V stars, would you expect them to enter a "common envelope" or mass transfer phase? Explain why or why not. (20 pts)

3. Galaxy Profiles

a.) Consider a spherical galaxy with a stellar density profile $n(r)$ which is viewed projected on the plane of the sky. Show that the projected surface density is given by,

$$\Sigma(R) = 2\int_0^\infty n(r)dz = 2\int_R^\infty \frac{n(r)rdr}{\sqrt{r^2-R^2}}$$

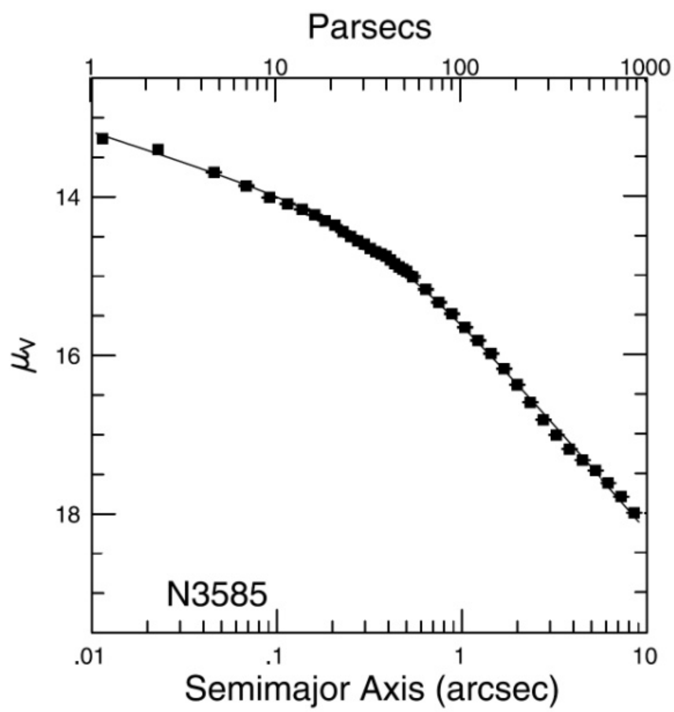
where *R* is the projected radius. (30 pts)

b.) Suppose that the density has a power-law form, $n(r) = n_0\left(\frac{r}{r_0}\right)^{-\alpha}$. Use the result from part (a) to show that for $\alpha > 1$,

$$\Sigma(R) = \Sigma_0\left(R/r_0\right)^{1-\alpha}$$, where $\Sigma_0 = 2I_\alpha n_0 r_0$ and $I_\alpha = \int_1^\infty \frac{x^{1-\alpha}dx}{\sqrt{x^2-1}}$

Discuss the physical significance of this result. (40 pts)

c.) The figure shows the surface brightness profile of NGC 3585 from HST observations. Determine inner and outer power-law slopes ($d\log\Sigma/d\log R$) and the corresponding inner and outer space-density slopes $(d\log n/d\log R)$ for this profile. (Note that the vertical axis is in magnitude units, rather than logarithmic units.) Assume that the mass-to-light ratio is independent of r. (30 pts)



4. Virial Theorem Applications

a.) Give a statement of the Virial Theorem and simple expressions for the potential and kinetic energies of an equilibrium stellar system of total mass M and characteristic radius R. Explain how the virial theorem can be used to estimate the total masses of stellar systems. What type of observations are needed for this application? For what type of systems is this approach most useful? (40 pts)

b.) Consider an isolated cluster of stars that is intially in virial equilibrium. Suppose that a fraction f of its mass is instantaneously removed at each radius, as a result of supernova explosions. By separately considering the instantaneous changes in the kinetic and potential energies that result from the mass loss, show that the system becomes unbound if $f > 0.5$. (40 pts)

c.) Assuming that the globular cluster remains bound, estimate the amount of time required for it reach a new state of virial equilibrium. You may use any resonable parameters for the mass and size of the cluster. (20 pts)

5. Radiative Shocks in the ISM

Whether or not a spherical blast wave due to a supernova is radiative or adiabatic is determined by comparing the expansion time with the radiative cooling time behind the leading shock wave of the blast. The following problem asks for background information on shocks and then requests an analysis of whether a shock will become radiative or not depending on the temperature sensitivity of the cooling rate.

a.) Write down (do not derive) the three Rankine-Hugoniot shock jump conditions for a plane-parallel adiabatic shock in an ideal gas with ratio of specific heats γ. Each of these relations expresses the conservation of a fundamental physical quantity. Label each jump condition accordingly. (30 pts)

b.) Now consider a spherical Sedov blast wave of constant total energy expanding into a uniform density background. Let t = 0 be the starting time of the blast at the origin of the spherical coordinates. For a Sedov blast, the shock speed as a function of time is given by $v = v_0(t/t_0)^{\alpha}$. What is the value of α for the type of Sedov blast described? Explain or derive this value. (25 pts)

c.) Assume that the shock is "strong" and explain what this means. Given the v(r) in part (b), explain why you expect the post-shock temperature T for the Sedov blast to obey the following relation: $T = T_0(t/t_o)^{2\alpha}$. (25 pts)

d.) Suppose now that the radiative cooling rate per unit volume of the post-shock radiative gas has the form $\Lambda \sim n^2T^{\beta}$. Suppose the blast starts out adiabatic (constant total energy as in parts [b] and [c]), in other words, the expansion time is initially much shorter than the post-shock radiative cooling time. Explain why the shock will then never enter a radiative phase if β is too large. What values of β are realistic? (20 pts)

6. The Eddington Luminosity

The Eddington Luminosity L_{Edd} is a concept that pervades modern astrophysics. The following problem asks you to derive the standard expression for L_{Edd} in the context of stellar evolution and then apply it to black holes.

a.) Write down the equations for hydrostatic and radiative equilibrium for a spherically symmetric star. (25 pts)

b.) Assume that the equation of state is dominated by radiation pressure and the opacity by electron scattering. Use the envelope approximation that $M_r = M$ and $L_r = L$ and derive an expression for L_{Edd}. Explain why, in spherical symmetry, L must be < L_{Edd}. (35 pts)

c.) Imagine gas accreting through the event horizon of a black hole of mass M and radius R at a rate dM/dt. Assume all the work done by gravity in reaching the event horizon is converted to luminous energy. Derive an expression for the dM/dt that produces a release of gravitational energy equal to the value L_{Edd}. This is called the "Eddington accretion rate". Evaluate the accretion time scale $M/(dM/dt)$ in years for a black hole accreting at the Eddington rate. (30 pts)

d.) The time scale in (c) presents difficulties for understanding how supermassive black holes formed so quickly in the history of the Universe. Discuss at least one way a black hole might be able to grow in mass by accretion at a rate greater than the Eddington accretion rate. (10 pts)

7. Why now?

One of the objections to the notion of Dark Energy is the question, Why now?

a.) The one 'natural' time scale in cosmology is the Planck time, in the sense that it is the only time that appears in the cosmological equations. It is the time that comes about with unit analysis by combining known physical constants in the Universe into a time. Write down an expression for the Planck time. Using the constants on the next page, evaluate your expression. What does the Planck time represent? (35 pts)

b.) The 'evolutionary' time scale in the universe is the Hubble time. What is the currently accepted value for Hubble constant? Convert this value for the Hubble constant into a time. Quite generally, the Hubble time is only a measure of the age of the universe in cosomogies without Dark Energy. Explain. The expression Hubble 'constant' is strictly speaking incorrect. *Simply* demonstrate this using the equations on the next page. (35 pts)

c.) Divide the Hubble time by the Planck time. Be careful about units. This number represents how long the Universe has been around when measured by a clock that moves (Flashes) at the natural Universal Rate defined by the Planck time. You should get a very large number. (10 pts)

d.) If Dark Energy turned on much earlier, say after a billion (10^9) or a trillion trillion (10^{24}) Flashes of the Universal Clock, we wouldn't be here to ask these questions because structure in the Universe would not have time to form. Why? Think evolution time scales on earth (geological, biological) in Planck times. If Dark Energy were to turn on much later than now, we would have never known about it. Why? (20 pts)