PH.D. QUALIFYING EXAMINATION

Department of Astronomy
August 24, 2006
9:00am - 1:00pm
[Ted—9:00am – 11:20am or 11:50am]

DAY ONE

Name: ______________________________

Student #: ___________________________

The exam sheets are inside this envelope and are not fastened together. Astronomy Program students MUST do the FIRST problem and SIX more problems from the remaining SEVEN. Astrophysics Program students MUST do the FIRST problem and THREE or FOUR more problems from the remaining SEVEN. When you are finished, please put the questions and your answer sheets back in the envelope in the correct order. Be sure the student number given you by the proctor is on every page of your answers.

Physical Constants

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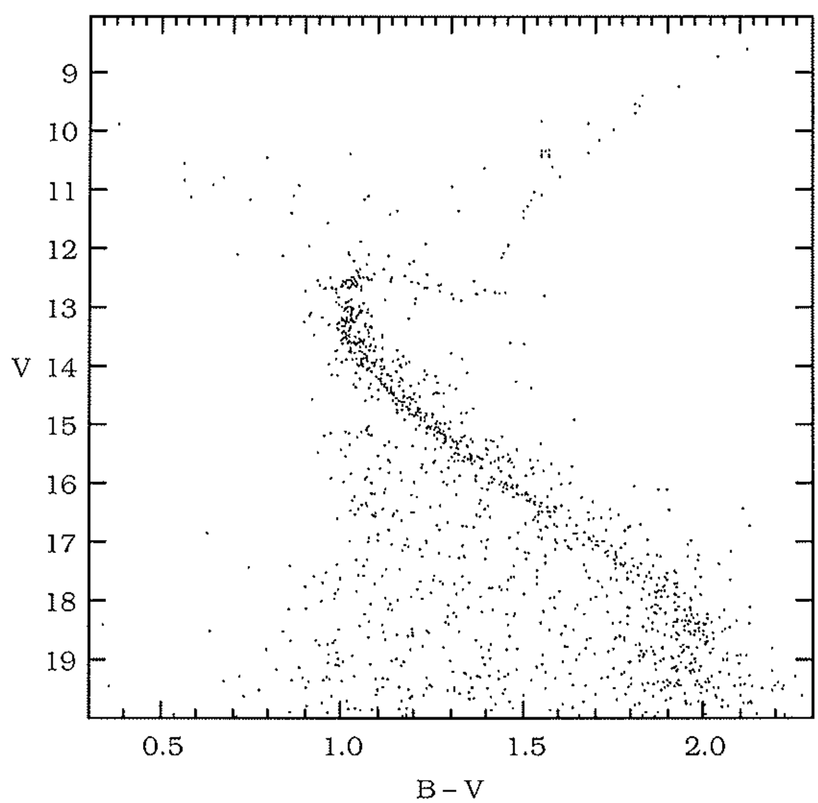
$k = 1.38 \times 10^{-16}$ erg/K
$m_p = 1.67 \times 10^{-24}$ g
$a = 7.56 \times 10^{-15}$ erg cm^{-3} K^{-4}
$e = 4.80 \times 10^{-10}$ esu
$m_e = 9.11 \times 10^{-28}$ g
$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} K^{-4} s^{-1}
$\sigma_T = 6.65 \times 10^{-25}$ cm^2

$R_\odot = 6.96 \times 10^{10}$ cm
$M_\odot = 1.99 \times 10^{33}$ g
$L_\odot = 3.90 \times 10^{33}$ erg/s
A.U. $= 1.50 \times 10^{13}$ cm
1 year $= 3.16 \times 10^7$ s
1 parsec $= 3.09 \times 10^{18}$ cm
$M_{V,\odot} = 4.83$ mag
B.C.($\odot$) = -0.07 mag
$(B-V)_\odot = 0.64$ mag
$T_{eff,\odot} = 5770$ k

1.) GENERAL ASTRONOMY (Mandatory Question)

This problem concerns 3 steps on the distance ladder: Cluster main sequence fitting, Cepheid variables, and Type Ia supernovae.

a) Below is a color-magnitude diagram of a particular direction in the Galactic disk that contains an open star cluster. It has been determined that: a) the reddening towards this cluster is $E(B - V) = 0.50$ mag and b) the cluster's composition is solar. Use the HR diagram below to estimate the cluster's distance. State your assumptions explicitly and show your approach clearly. (25 pts)



b) Explain the underlying astrophysics behind (25 pts)

(i) Cepheid variability and the period-luminosity relationship
(ii) Type Ia supernovae

c) The accuracy of each of these steps on the distance ladder is limited by ambiguities and second-order effects. Choose ONE of the techniques listed at the top of the page and describe the ambiguities inherent in the process. Then, design an observational program that could help further elucidate one of the second-order effects you have described. (50 pts)

*2.) Noise

a) When the sky background is low, CCD readout noise can become an important noise source. This is often the case for high dispersion spectroscopy. Demonstrate numerically that the S/N can be a sensitive function of read noise, but relatively insensitive to background, using the following three examples. Assume that the number of detected photons is 200 per resolution element.

Read noise in rms electrons	Detected background photons
5	10
5	30
15	10

Using the numbers in the table, what is the percentage decrease in S/N as the background is increased by a factor of 3 with constant read noise? What is the percentage decrease in S/N as the read noise is increased by a factor of 3 with constant background? (35 pts)

b) For most low dispersion spectroscopy the opposite is true (i.e., S/N is much more sensitive to background than to readout noise). Explain why this is the case. (25 pts)

c) Describe the major contribution(s) to the sky background for
   i) 2000 Å at low galactic latitude, from space
   ii) 8000 Å at new moon, from the ground
   iii) 20-microns, from the ground
   iv) 20-microns, from space

What property of (ii) above makes the background particularly sensitive to spectral resolution? (40 pts)

3.) Behavior of Spectral Lines

a) The Balmer lines of hydrogen reach maximum strength among the A stars. Explain why the lines are weaker in cooler stars and in hotter stars. (25 pts)

b) Spectral lines of both neutral and ionized metals become weaker as one moves up to higher temperature from the G and K stars, even as the dominant ionization state shifts from neutral to singly ionized. Why? (25 pts)

c) What factors determine how strong a spectral line will be? What factors determine how wide a spectral line will be? (25 pts)

d) Why do the resonance lines of alkali metals become very strong in the spectra of very low mass dwarf stars? (25 pts)

4.) Dark Matter/Energy

a) What is $\Omega_{total} = \rho_0/\rho_{crit}$ thought to be today (August 2006)? What geometry does this imply? Discuss specific experimental evidence supporting this value (e.g., WMAP, Type Ia supernovae, cluster mass determinations). What fractions of the total energy density in the universe are baryonic, nonbaryonic, and dark energy? (40 pts)

b) *Dark Matter*. There are really two dark matter problems: (1) baryonic dark matter and (2) nonbaryonic dark matter. What fraction of the baryonic matter is *visible* or observed baryonic matter? What is the evidence for baryonic dark matter in excess of visible matter besides global fits to WMAP data? What is the evidence for nonbaryonic dark matter? (30 pts)

c) *Dark Energy*. What is the experimental evidence for dark energy? Schematically diagram the evolution of the scale factor R(t) in the absence of dark energy for k = -1, 0, +1. Derive the evolution of the scale factor R(t) when dark energy dominates. For k = 0, sketch Hubble's law (log cz vs m) in the presence of dark energy, particularly at large z. Explain. What is dark energy? (30 pts)

5.) Planetary Atmospheres

Recall that the equation of hydrostatic equilibrium can be written:

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

a) Give a derivation of the equation of hydrostatic equilibrium by taking into account the balance of gravitational and pressure forces on a volume of gas located at a distance r from a gravitational force center. (50 pts)

b) What is meant by the isothermal approximation for a planetary atmosphere? Consider the limit of a thin plane parallel atmosphere. Show that the use of the isothermal approximation with the ideal gas law results in an exponential solution to the equation of hydrostatic equilibrium for a planetary atmosphere of the form:

$$P(z) = P(0)e^{-z/h},$$

where z is height above the planetary surface and h is the scale height. Show that the constant h can be expressed in terms of the temperature T and mean molecular mass μ of the atmosphere, and the mass M and radius R of the planet. (50 pts)

6.) The Main Sequence

a) Describe how the properties of stars vary along the main sequence from the lowest to highest mass stars. Your answer should include all of the following items: (50 pts)

- gas characteristics: equation of state, opacity sources, nuclear reactions
- internal structure: convective and radiative regions, typical central densities and temperatures
- mass M, radius R, luminosity L, and effective temperature T_{eff}

b) Use dimensional analysis of the relevant stellar structure equations to show that

$$L \propto M^3$$

along the main sequence when you use an ideal gas equation of state and assume that the mass absorption coefficient is independent of temperature and density. (50 pts)

*7.) King Models

The distribution function for a King model of a spherical star cluster has the form,

$$f_k(\varepsilon) = \begin{cases} c_k[\exp(\varepsilon/\sigma^2) - 1] & \varepsilon > 0 \\ 0 & \varepsilon \le 0 \end{cases}$$

where the relative energy is $\varepsilon = \psi - \frac{1}{2}v^2$, the relative potential is $\psi = \Phi(r_t) - \Phi(r)$, with r_t the tidal radius, and c_k and σ are constants.

a) Explain the physical motivation for this form of the distribution function. Why is it a solution to the collisionless Boltzmann equation? Discuss the role of the two terms in the square brackets. What difficulty results if cluster models are generated without the second term? (40 pts)

b) Rewrite f_k as a function of v and v_{esc}, where the latter represents the escape velocity to the tidal radius. On one set of axes, sketch the dependence of f_k on v for the following two locations in a cluster for which the central escape velocity is large compared with the rms velocity:

(i) deep in the core, near the cluster center
(ii) in the halo, a small distance inside of r_t

Indicate the approximate location of σ on your velocity axis. Use your sketches to argue that $\langle v^2 \rangle$ approaches the isothermal value ($3\sigma^2$) in the core but that $\langle v^2 \rangle \ll 3\sigma^2$ in the outer halo. (40 pts)

c) Sketch the $\rho(r)$ density profile on a log-log plot. Indicate the location of the core radius and the tidal radius. How does a King model differ from an isothermal sphere? (20 pts)

8.) Neutral Hydrogen

a) Consider a 2 level atom. Assuming the gas is in local thermodynamic equilibrium (LTE), write down the relationship between the observed number densities of the two populations and the energy difference of the two states, multiplicities, and kinetic temperature of the gas. (20 pts)

b) The Einstein coefficient for spontaneous emission of the 21 cm spin flip transition in the ground state of neutral hydrogen is $A_{21} = 2.85 \times 10^{-15}\ s^{-1}$ and the ratio of multiplicities for these two hyperfine states is 3.

(i) Using your relationship from (a) above, and the assumption that all of the hydrogen gas is in one of these two states, derive the total number of 21 cm photons emitted per unit time per unit volume as a function of the number density of hydrogen.

(ii) Does this result depend strongly on the kinetic temperature of the gas? Explain your answer using temperatures appropriate for typical Galactic locations of neutral hydrogen. (40 pts)

c) Assume again that the 21cm line is in LTE. Assume the medium you are observing is uniform. Ignore background sources and Doppler shifts. Write down an expression or expressions which relate the brightness temperature T_b of 21cm emission to the column density of atomic hydrogen and the kinetic temperature of the gas. Under what conditions can you determine the kinetic temperature from T_b? Under what conditions can you determine the column density? (40 pts)

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{\exp(hc/\lambda cT) - 1}$$

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

$$\frac{\dot{R}^2}{R^2} + 2\frac{\ddot{R}}{R} + \frac{8\pi G p}{c^2} = \frac{-kc^2}{R^2} + \Lambda$$

$$\frac{\dot{R}^2}{R^2} - \frac{8\pi G\rho}{3} = -\frac{kc^2}{R^2} + \Lambda/3$$

$$H = (\frac{\dot{R}}{R})$$

$$q = -\frac{\ddot{R}}{RH^2}$$

$$\rho_c^0 = \frac{3H_0^2}{8\pi G} = 1.88\times 10^{-29} h^2 \text{gm/cm}^3$$

$$\epsilon_\gamma = a_B T^4 = 3p_\gamma$$

$$p = \omega\rho c^2$$

$$\frac{R_0}{R} = 1 + z$$

$$H^2(z) = H_0^2[\Omega_m^0(1+z)^3 + \Omega_r^0(1+z)^4 + \Omega_v^0(1+z)^{3(1+w)} + (1-\Omega_T^0)(1+z)^2]$$

$$R_0 r(z) = \frac{2c}{H_0\Omega_0^2(1+z)}[2-\Omega_0+\Omega_0 z - (2-\Omega_0)(1+\Omega_0 z)^{\frac{1}{2}}],\ (MD, \Omega_0 = \Omega_M^0, \Lambda = 0)$$

$$\varepsilon_{rel} = \rho_{rel}c^2 = \frac{\pi^2}{30} g_* T^4 \times k_B^4/(c\hbar)^3 = 3p_{rel}$$

$$s = \frac{2\pi^2}{45} g_{*s}(k_B T/\hbar c)^3$$

$$\text{eV} = 1.66 \text{ x } 10^{-12} \text{ erg} = 1.66 \text{ x } 10^{-19} \text{ J}$$

$$T_0 = 2.73\text{K}$$

$$s_0 = 2970 \text{ cm}^{-3}$$

PH.D. QUALIFYING EXAMINATION

Department of Astronomy
August 25, 2006
9:00am - 12:30pm
[Ted—9:00am – 10:50am or 10:20am]

DAY TWO

Name: ______________________________

Student #: __________________________

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$\sigma_T = 6.65 \times 10^{-25}$ cm^2	$T_{eff,\odot} = 5770$ k

1.) Real Stars

a) Estimate the mass of the nearby star Arcturus *in solar units* assuming a temperature and surface gravity of 4300K and log g = 1.0. The distance to Arcturus is 11.26 parsecs, as measured using the Hipparcos satellite. Its apparent visual magnitude is V=-0.05. Clearly state all the assumptions you make. (50 pts)

b) Comment on your estimate. In particular, is your result physically realistic? Describe the results you might expect if your assumptions are incorrect. (50 pts)

*2.) Nuclear Reactions in Stars

Carefully explain why thermonuclear reactions in stars are so temperature sensitive by discussing the various terms in the reaction rate integral. What parameters cause some reactions to be more temperature sensitive than others? What are the approximate temperature dependencies of the proton-proton chain, of CNO hydrogen burning, and of helium burning? Write down the reactions of the CN cycle and explain how the operation of the CN cycle in "equilibrium" burning affects the relative abundances of ^{12}C and ^{14}N. (100 pts)

3.) Active Galaxies

a) Derive the Eddington Luminosity for a black hole of 3×10^6 solar masses. Compare this to the total luminosity of the Milky Way. What would an outside observer see if the center of our Galaxy became an active galactic nucleus? (50 pts)

b) To appreciate the observational difficulties associated with trying to detect a galaxy's stellar emission around a distant quasar, calculate the apparent magnitude (ignore the k-correction) and angular size that a large, luminous galaxy (such as the Milky Way) would have at a redshift of (i) $z = 0.1$; (ii) $z = 1.0$. You may adopt any reasonable cosmology for this question. (50 pts)

4.) One Degree Imager

The WIYN One-degree Imager (ODI) will consist of a 64 x 64 array of independently-clocked orthogonal-transfer array (OTA) CCDs. Each CCD can be used for either long integration or for fast guiding during a long integration. The fast guiding will be used to correct for first order (tip-tilt) atmospheric seeing, in order to improve the angular resolution.

a) How does the clocking of an OTA differ from the clocking in a conventional CCD? (30 pts)

b) Why are multiple CCDs used for fast guiding, instead of just one? (20 pts)

c) Higher angular resolution from the ODI will still leave some close pairs of star unresolved. Consider two unresolved stars having V = 17, B-V = 0.0 and V = 18, B-V = 2.5. What is the V and B-V of the blended pair? How could you distinguish such an unresolved pair from a single normal star using photometry? (30 pts)

d) Describe how you would take advantage of the properties of the ODI in devising a research program. (20 pts)

5.) Stellar Atmospheres

a) What is the dominant source of opacity at visible wavelengths in the atmospheres of O stars? A dwarfs? K giants? (30 pts)

b) Identify four different physical characteristics that can be used to specify a "temperature" of a stellar atmosphere. For each physical process, what observations would you use to determine temperature? (40 pts)

c) Assuming a blackbody spectrum, calculate the wavelength at which F_ν is greatest for the Sun and the wavelength at which F_λ is greatest. Explain why these are not the same. (30 pts)

6.) HII Regions

Suppose a dwarf galaxy has a metallicity three times lower than that of the Sun. How might this affect the temperature of its HII regions compared with temperatures typical of HII regions in our own Milky Way? Your answer should include a thorough and precise explanation of how HII regions are heated and cooled.

(100 pts)

*7.) Mass Segregation

a) Describe what is meant by mass segregation in a stellar system. Explain why the tendency toward energy equipartition leads to mass segregation. When is mass segregation expected in a stellar system? (30 pts)

b) The Jeans equation for a spherical stellar system with an isotropic velocity distribution may be written,

$$\frac{d(n\sigma^2)}{dr} = -n\frac{d\Phi}{dr},$$

where n is spatial density and σ is one-dimensional velocity dispersion. Describe the derivation of this equation from the collisionless Boltzmann equation; do not do the derivation. (20 pts)

c) Consider two different mass groups of stars present in a gravitational potential $\Phi(r)$, with masses M_1 and M_2 ($M_2 > M_1$). Assume that the velocity distribution for each group is isotropic with a constant velocity dispersion. Also assume that there is energy equipartion between the two groups. Use the Jeans equation to show that:

$$n_2(r) \propto [n_1(r)]^q \text{ where } q \equiv M_2/M_1.$$

Discuss why this result implies mass segregation in the expected sense. (50 pts)

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